



TECHNICAL REPORT NO. 10

Horizontal Accuracy of Digital Elevation Models

M. Nour Hawa Thomas Knudsen Simon L. Kikkendorff Brian P. Olsen Brigitte C. Rosenkranz

DANISH MINISTRY
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M. Nour Hawa Thomas Knudsen Simon L. Kokkendorff Brian P. Olsen Brigitte C. Rosenkranz: Horizontal Accuracy of Digital Elevation Models

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Preface

Lolita Bahl

Quality control procedures for digital elevation models are often focused on checking the validity of the raw measurements behind the final gridded model. And rightfully so since, as the adage goes, “*garbage in, garbage out*”: without good measurements it is impossible to obtain a good final model.

But for the end user of the gridded model, the validity and accuracy of the model *per se* is much more interesting. And while the vertical accuracy of the final model can be checked in a fairly straightforward manner (by comparing independently surveyed point heights with height values obtained by interpolation in the grid), the same is not true with respect to the horizontal accuracy: The horizontal accuracy relates to the true position of objects in the landscape – objects which may even be hard to resolve due to the limited grid cell size.

For DK-DEM, the new Danish national elevation model, we have a stated goal of a horizontal accuracy on the order of 1 m – but a grid cell size of 1.6 m. It should be evident that while we have all reasons to believe that the desired accuracy has been reached for the LiDAR point cloud behind DK-DEM, it is an entirely different *and much more complex*, story to ensure that the accuracy has been carried all the way through to the grid.

The following papers tell that “much more complex story”: First an executive summary, confirming that the accuracy goal has been reached. The summary is followed by a paper describing *in which sense* we define horizontal accuracy. The third paper describes how the actual tests were carried out, and is followed by 4 lab reports describing the results from each of 4 test sites.

DK-DEM is a large and complex dataset, so by its very nature it *will* have occasional errors. The present report does, however, support the assertion that the overall quality of DK-DEM is very high.

I would like to thank all the authors for their efforts in documenting this. Special thanks go to M. Nour Hawa for his excellent work, carried out during an internship with KMS as part of his MSc (surveying) studies.

Copenhagen, December 2010

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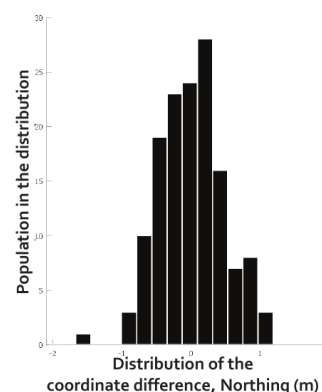
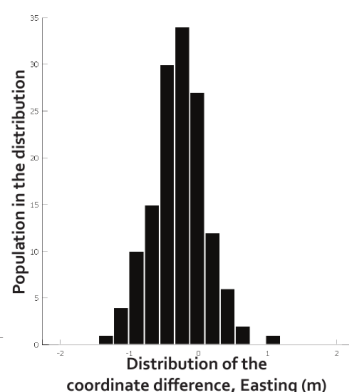
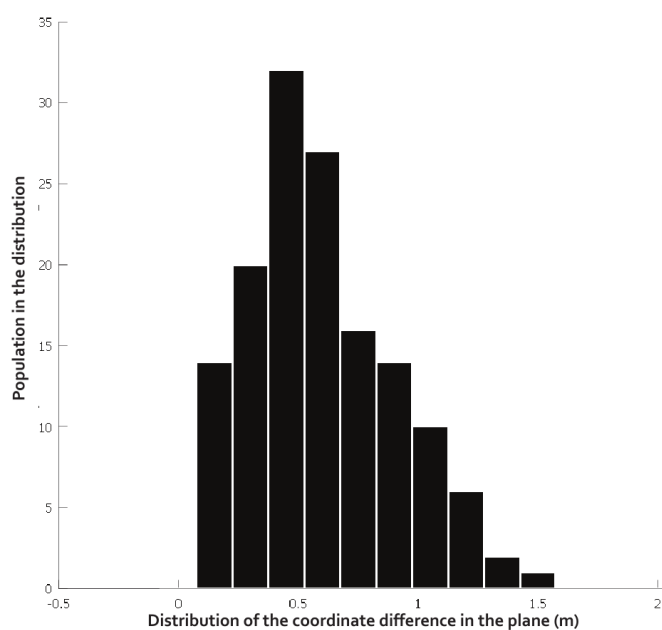
Horizontal Accuracy of DK-DEM: Summary of results.

M. Nour Hawa (Aalborg University) and Thomas Knudsen (Kort & Matrikelstyrelsen).

| Test site results | Aalborg | Århus | Esbjerg | Nordsjælland |
|----------------------|----------------|----------------|--------------|--------------|
| Reference data | Scankort/TOP10 | BlomInfo/TOP10 | BlomInfo/FOT | Scankort/FOT |
| Number of points | 160 | 136 | 132 | 140 |
| RMSE, plane | 82 cm | 64 cm | 55 cm | 60 cm |
| Easting, Mean Error | -47 cm | -14 cm | -23 cm | -25 cm |
| Northing, Mean Error | -7 cm | 6 cm | 8 cm | -4 cm |

Combined results

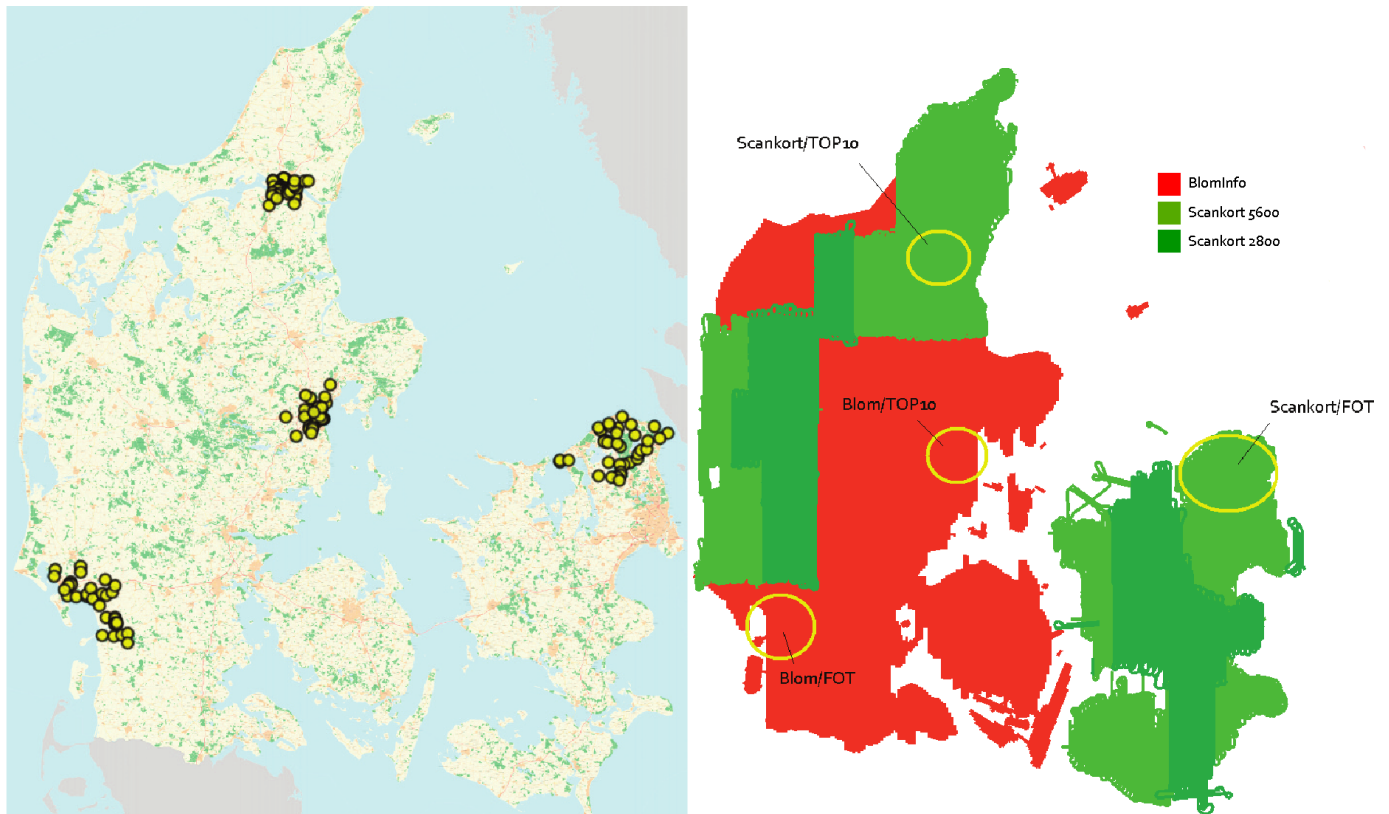
| | |
|----------------------|-------------------------------|
| Number of points | 568 |
| RMSE, plane | 67 cm |
| Easting, mean error | -28 cm $\sigma_E = \pm 39$ cm |
| Northing, mean error | 0 cm $\sigma_N = \pm 47$ cm |



The figure above summarizes the results from a series of estimates of the horizontal accuracy of DK-DEM, the national coverage Danish elevation model. The accuracy was estimated at four representative test sites (Aalborg, Århus, Esbjerg, and Nordsjælland), through comparison of building corners as measured in the elevation model, with corresponding coordinates as registered in two topographic databases (FOT and TOP10DK).

The results indicate that the grid production work flows introduce small biases in the easting accuracy, and somewhat smaller biases in the northing accuracy. But evidently from the RMSE values, the biases do not influence the accuracy in any intolerable way: **The horizontal RMSE ranges from 0.55 m to 0.82 m, for an overall RMSE of 0.67 m. This is well below the original requirement of 1.0 m.**

Note however, that the question of the horizontal accuracy of a grid, is somewhat ill-posed. So before attempting to interpret these figures in any extensive way, please consult the papers accompanying this summary, where we present the details of the estimation method, and give the rationale for its use.



The left panel of the figure above shows the actual positions of the buildings measured. The right panel shows the approximate positions of the test sites, and the combinations of data providers (BlomInfo, Scankort), laser scanners (Riegl Litemapper 2800, Riegl Litemapper 5600, Optech ALTM3100), and topographic databases (FOT, TOP10DK).

The emphasis has been on testing each possible combination of data provider and topographic database. This should reveal any effects from the differing work flows of the providers, while also testing the estimation method by comparing results based on the older TOP10DK data with results based on the newer and presumably more accurate FOT data. As expected, TOP10DK data agrees very well with DK-DEM, while FOT data tends to agree even better.

Points worth noting:

1. The results from the Aalborg test site are well within the limits set by the original specification, but notably worse than results from the three other test sites. Further investigation has indicated that this is due to a site specific variation in the kind of ground control points available as reference material for the original adjustment of the laser point cloud.
2. Sites measured with the Riegl 2800 scanner have not been tested independently. The 2800 and 5600 data were, however, processed through the same work flow (by Scankort) and integrate seamlessly at the boundaries.
3. Bornholm is not represented in the test. But Bornholm was measured and processed by BlomInfo using the same Optech ALTM3100 setup as in the two existing BlomInfo test cases.

All in all, the test results give us good reasons to believe that the overall horizontal accuracy of DK-DEM is well within the stated requirement of 1 m.

On the Horizontal Accuracy of Digital Elevation Models

Thomas Knudsen, Simon L. Kokkendorff, Brian P. Olsen, Brigitte C. Rosenkranz

Abstract

We give the rationale behind a simple method for determination of the horizontal accuracy of a raster elevation model, and present a brief and informal analysis of why this metric is ill-posed unless further constraints and definitions are introduced.

Keywords: Digital surface model, digital elevation model, raster, compressive sampling, horizontal accuracy, quality control, LiDAR

1 Introduction

In the early spring of 2010 we were asked to derive a measure for the horizontal accuracy of DK-DEM, the new national coverage Danish elevation model.

DK-DEM consists of gridded data with a node length of 1.6 m. Colloquially this is often referred to as *the resolution* although in classical terms, the resolution really is twice that number. To avoid any confusion we will use the remote sensing term GSD, *Ground Sample Distance* below.

The specification behind the original tender for DK-DEM required a horizontal accuracy of 1 m. While this kind of requirement is well defined and makes good sense for the LiDAR point cloud behind the gridded model, it is an entirely different thing for the gridded model itself. We will give an informal analysis of this in section 2 below, but assume that the reader will readily accept that the concept of a 1 m accuracy for a grid with a GSD of 1.6 m is non-trivial to define rigorously.

2 Why is the *horizontal accuracy of a gridded data set* an ill-posed question?

In order to answer the question of why the accuracy of a gridded data set is an ill-posed question, we will first frame it by describing why the complementary question of the horizontal accuracy of a corresponding LiDAR point cloud, is a *well posed* question.

This question is well posed because for every LiDAR reflection in the point cloud, there has been a corresponding reflector on the ground. Hence, we may in a meaningful way compare the measured *northing, easting, height* (NEh) triplet, with the surveyed position of an actual physical object on the ground. If not always in practice, then at least ideally speaking.

This should also give the first hint at why the original question is ill-posed: *for a gridded data set there is not necessarily any real world object corresponding to the height registered in*

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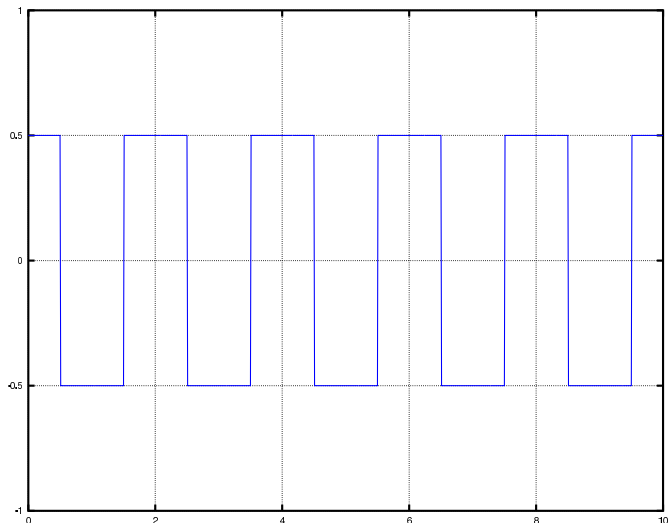


Figure 1: Five cycles of a square wave. In the following figures, we zoom in on one of these cycles for a very simplified building model.

a given grid node! To evaluate the horizontal accuracy, we must look at large, well defined structures in the landscape (rather than well defined *points* as specified in accuracy standards such as ASPRS (2005); FGDC (1998)). In the urban landscape, such structures are ample since block shaped buildings are littered literally at every street corner. So in order to gain some insight in the problem, we take a brief look at a very simple building model.

2.1 Representing a block shaped feature in an elevation model

To understand the characteristics of a block shaped feature, we initially model the feature as one full cycle of a square wave (cf. figure 1).

Following the (time domain) notation of Smith (1997), we then write up the Fourier synthesis of the wave as

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi ftn) - \sum_{n=1}^{\infty} b_n \sin(2\pi ftn) \quad (1)$$

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt \quad (2)$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos\left[\frac{2\pi tn}{T}\right] dt \quad (3)$$

$$b_n = -\frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin\left[\frac{2\pi tn}{T}\right] dt \quad (4)$$

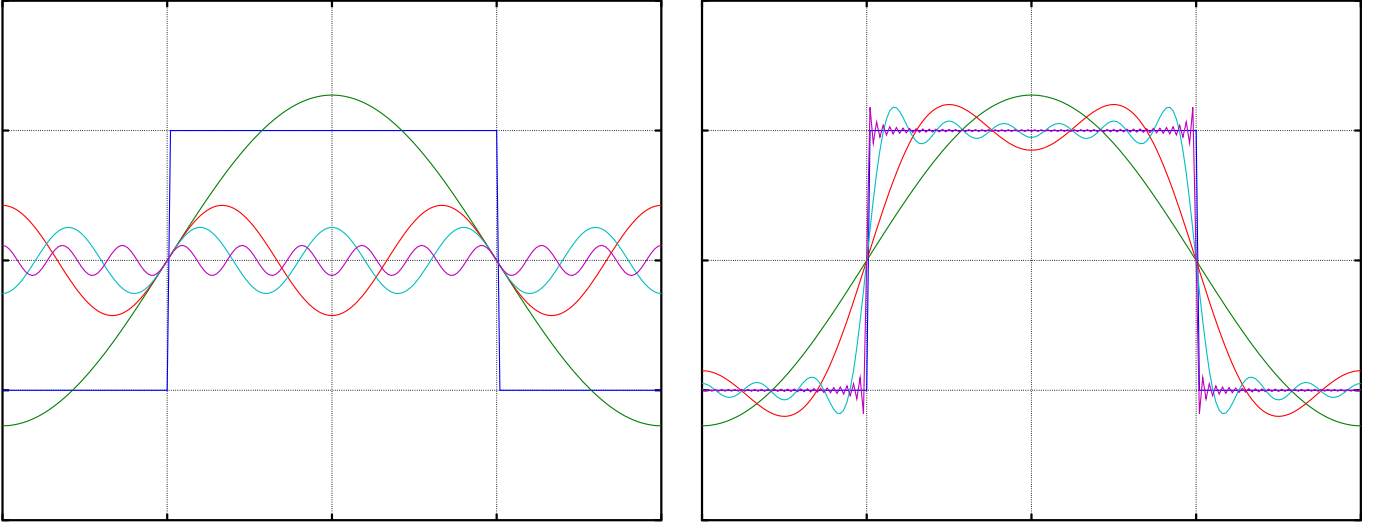


Figure 2: A building model created by zooming in on one cycle of the square wave from figure 1 above. The *left panel* shows the square wave and its Fourier cosine components number 1, 3, 5, and 11. While the amplitude of the components diminish as the frequency rises, the components still contribute significantly to the series through the cumulative effect. This is evident from the plot in the *right panel* where we show the same square wave, now accompanied by its Fourier cosine series, truncated after component number 1, 3, 11, and 99, respectively. In order to represent the flat top of the wave (i.e. the wall/roof transition of a building) realistically, we need to sum the series up to a rather high frequency.

Where T is the period of the square wave (corresponding to its wavelength in spatial terms). In our case of a square wave with a pulse height (A) of unity, this simplifies to

$$a_0 = 0 \quad (5)$$

since the square function is symmetric and hence vanishes when integrated over a full period; additionally

$$b_0 = 0 \quad (6)$$

since the square function is symmetric wrt. $t = 0$, whereas the sine components are antisymmetric, and finally

$$a_n = \frac{2}{n\pi} \sin \frac{n\pi}{2} \quad (7)$$

which vanishes for even n , so we are left with a series consisting of odd-numbered cosine components reducing equation 1 to the much simpler

$$x(t) = \sum_{n=1,3,\dots}^{\infty} \frac{2}{n\pi} \sin \frac{n\pi}{2} \cos(2\pi ftn). \quad (8)$$

In figure 2 we demonstrate that a number of high frequency components is needed in order to represent a wall/roof transition in any realistic way. From the figure, it is evident that we need *at least* all components up to 11 in order to have a reasonable (but still blurry) representation of the building. What does that mean for our ability to locate buildings in the elevation model data?

2.2 So what?

First and foremost let us quantify a bit: Assume that a typical building has a cross section of 10 m. In that case, the fundamental wavelength of our Fourier synthesis will be twice that, i.e. 20 m. The eleventh cosine component will have a wavelength of $20 \text{ m} / 11 \approx 1.8 \text{ m}$. This is very close to the 1.6 m

GSD of the grid. But according to the Nyquist/Shannon sampling theorem (Koopmans, 1974; Shannon, 1948, 1949; Smith, 1997) a sampling with a given GSD will only resolve signals with wavelengths longer than twice that GSD. So in our case of a 1.6 m GSD, we cannot resolve wavelengths shorter than 3.2 m, which is significantly longer than the 1.8 m required to represent an even moderately sharp edge.

2.3 But...

Nevertheless, it is well known from photogrammetry that during the orthorectification of aerial photos, even a non-skilled stereo analyst is able to pinpoint the location of well defined points (e.g. road crossings, building corners, etc.) with an accuracy that is 3–5 times better than the GSD of the photos (Geoforum Danmark, 2005).

In the light of the analysis above, this is quite counterintuitive. But from everyday experience it is not at all surprising, since the stereo analyst is able to incorporate a large amount of *a priori* knowledge and experience in the interpretation. Especially it is not surprising that the corner of a building (defined as the intersection between lines secant to two perpendicular walls) is much better defined than any individual point *on* the walls.

In recent years, the theoretical basis for such phenomena has been enhanced through the research field of *compressed sampling* (Candès, 2006; Candès and Wakin, 2008).

3 Method

Utilizing these analyses, we proposed to evaluate the accuracy of the gridded data by following these steps:

Compute nDSM. The *normalized digital surface model* (nDSM) is defined as the difference between the surface model (DSM) and the terrain model (DTM), i.e. $nDSM = DSM - DTM$. The nDSM highlights blob shaped objects

in the landscape (buildings, trees), simplifying the next steps.

Threshold nDSM. Threshold the nDSM at the ≈ 2 m level. This should highlight buildings even more.

Digitize building corners. Display the nDSM in a GIS, and digitize the corners of a number of buildings.

Compare with known data Compute the accuracy as the (RMS sense) difference between the digitized corners and the existing registrations from e.g. map databases.

As outlined above, the algorithm seems deceptively simple. But as in all cases of real world data investigation, a large number of devils lurk in the details. These devils were chased, captured and crunched by M. Nour Hawa, who did the actual implementation of the algorithm, and subsequently, carried out the investigation of the accuracy of DK-DEM, as reported in Hawa (2010a,b,c,d,e); Hawa and Knudsen (2010).

3.1 But does this really measure the accuracy?

How can we be sure that the measure arrived at actually tells something meaningful about the accuracy of the elevation model? After all, the registration variance of a typical map database is on the same order of magnitude as the accuracy we want to determine. We argue that the measure is meaningful through the statistical fact that (under appropriate assumptions) *the variance of the difference between two elements is greater than the particular variances of the elements*

To see this, we first assume that the absolute error of point coordinates in a collection of geodata (i.e. the difference between the point coordinates and the coordinates of the corresponding “true physical” points) are samples of a random distribution with some standard deviation, which reflects the quality of the dataset.

Under this assumption the relative error between two different, *uncorrelated* datasets, which describe the same physical points, will be randomly distributed with a standard deviation which is *greater than the individual standard deviations*. This follows from the basic addition formula for variances:

We let A and B denote the two measured datasets, and let C_A and C_B denote the point coordinates (say, the easting coordinates). Then the absolute errors of these coordinates are given as $E_A = C_A - C_{truth}$ and $E_B = C_B - C_{truth}$.

From this we may compute the *relative error* between the coordinates in the two datasets as:

$$\begin{aligned} \text{Var}(C_A - C_B) &= \text{Var}(C_A - C_{truth} - C_B + C_{truth}) = \\ \text{Var}(E_A - E_B) &= \text{Var}(E_A) + \text{Var}(E_B) - 2\text{Cov}(E_A, E_B) \Rightarrow \\ \text{Var}(E_A - E_B) &\approx \text{Var}(E_A) + \text{Var}(E_B) \end{aligned}$$

Or in other words: under the assumption of no correlation between the two elements, *the variance of the difference is larger than the largest variance of the individual elements*.

4 Caveats

There are however a few caveats with respect to the algorithm. First and foremost we need buildings (or other large scale rectangular features). Hence, this kind of accuracy estimation can only be carried out in relatively densely populated areas (i.e. cities and suburbs). Second, we need to substantiate our assumption of no correlation between the errors in coordinates

obtained from map data and coordinates obtained from DK-DEM.

Essentially, the map data are derived from photogrammetric stereo measurements. Most often, this is done using frame data with direct georeference from GPS/INS, further refined using block adjustment based on a large number of well defined points measured in the photos.

The DEM data on the other hand, is derived from a LiDAR point cloud. While still using the GPS/INS combination for georeference, the scanning geometry of the LiDAR data acquisition system is very different from the frame geometry of the photogrammetric camera.

Hence, the production workflows between the map data and the DEM are very different, justifying the assumption that the same is true with respect to the primary error sources for the two data sets. This is also almost certainly true in the case of the DK-DEM terrain model. The DK-DEM *surface model* was, however, improved by applying three different rules derived from existing building registrations (Flatman, 2010):

1. The DSM interpolation routine was set to allow sharp steps in height within a buffer zone derived from existing registrations.
2. The DSM data rejection criterium was relaxed within the polygons of the existing building registrations.
3. The DSM interpolation routine was allowed to do long baseline interpolation (and extrapolation) *within the domain* defined by a building registration.

Item 1 above allows the interpolation routine to actually produce the sharp steps in height that are essential in order to represent a building in the DSM (in a DTM, sharp steps are usually indications of inadequate removal of above-terrain points in the point cloud). But the interpolation routine will only produce the steps *if point data actually support that they are present*.

Item 2 above was introduced in order to be able to represent buildings with very dark roofs (primarily roofing felt): in such cases it is commonly seen that only a few points are actually reflecting light back to the LiDAR instrument. Under normal statistical assumptions, such observations will be discarded as outliers. Using the item 2 criterion, it was possible to arrive at a much more accurate model by keeping these data, and applying the item 3 criterion to the local interpolation (essentially equivalent to filling in with a mean observed height inside sparsely measured building outlines).

Hence in order for the *horizontal accuracy algorithm* to work properly, it is essential that it is used on datasets based on measurements from well reflecting roofing material only. In practice, this means that the interpretation must be supported by a colour orthophoto for the selection of light, well reflecting roofs.

5 Conclusion

In section 1 above, we assumed the reader would readily accept that the concept of a 1 m accuracy for a grid with a GSD of 1.6 m was non-trivial to define rigorously.

In conclusion we would like to stress that we have certainly not given a rigorous definition of this ill posed question. We have however, given a number of good reasons for the plausibility of the algorithm proposed.

The practical implementation and test by Hawa (2010a,b,c,d,e), supplements the plausibility by indicating the practical usefulness of the algorithm.

But further analysis and scrutiny of the subject would certainly be a matter of more than (just) academic interest.

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A Method for Assessment of the Horizontal Accuracy of a Digital Elevation Model

M. Nour Hawa

Abstract

This article introduces a simple method for determination of the horizontal accuracy of a raster digital surface model (DSM) and corresponding digital terrain model (DTM). The method (which is implemented using readily available software) is based on determination of coordinate differences between reference data and measured objects in the DSM. Measuring objects in the raster model is the challenge. The method is presented in general terms, and applied in a case study of the official Danish height model, DK-DEM.

Keywords: Digital surface model, digital elevation model, raster, compressive sampling, horizontal accuracy, LiDAR

1 Introduction

Often, when validating and verifying digital elevation models (DEMs), only the vertical accuracy is considered—probably because the DEM is typically created from aero-triangulated LiDAR data, that have already been evaluated for horizontal accuracy.

But even in the case of perfect horizontal matching, the DEM may still suffer from errors introduced during interpolation and rasterization of the point cloud. Verification by vertical accuracy only, yields results which are only strictly valid on level terrain. Due to the unknown horizontal accuracy, it is impossible to gain any significant insight into the vertical accuracy of the model in sloping areas.

Unlike an orthophoto, object features other than height cannot be seen directly in a DEM. Hence, in the case of DEMs we cannot use the well established methods for assessment of horizontal accuracy of orthophotos (cf. e.g. Geoforum Danmark, 2005). Generally a DEM is also of coarser resolution than an orthophoto. This makes it troublesome to determine where the reference objects/ground control points are to be measured.

In section 2 below, we present a simple method for validation and verification of the horizontal accuracy on a DEM (or in more specific terms: a digital terrain model, DTM, respectively a digital surface model, DSM). In section 3, a case study will be shown using this method to assess the accuracy of the official Danish height model, DK-DEM.

2 Method

In broad terms, the method described below, is based on manual measurement of reference object coordinates in the DEM and comparison with suitable reference data.

Due to the typical resolution of DEMs, buildings are the only geometrically well defined objects that can be large

enough to be clearly visible in the DEM. Hence, we choose buildings as our target objects.

The method is presented in diagrammatic form in appendix B below. The description here is mostly comments elaborating somewhat on the diagram.

The main steps of the algorithm are:

1. Prepare the DEM data
2. Find and measure the objects
3. Compare with reference data to obtain the results

The reference data consists of coordinates of the building corners. These can be obtained from field surveys, or from a suitable map product. The accuracy of the reference data should be taken into account.

To optimize the measurement conditions, a normalised DSM (nDSM) is created, by subtracting the DTM from the DSM. Then a vertical threshold is selected, so only objects higher than the threshold value is visible (see figure 1).

The next step is to find suitable buildings and measure them. With the nDSM it is possible (even easy) to identify buildings. The measurement is carried out by visual estimation: it is well known from photogrammetry, that even an unskilled stereo operator is able to estimate the position of rectangular crossing lines with an accuracy that is 3–5 times better than the resolution of the imagery (e.g. the réseau marks of traditional metric cameras utilize this by design).

The mismatch of each corner of a building is combined into one value, representing the displacement for each building measured. The overall characteristics for each test site is obtained as the primary statistical descriptors (mean, variance, etc.) for a number of buildings.

2.1 Interpretation and measurement

The main problem of the method is to determine how to measure with sufficient accuracy. The resolution/grid size plays a significant role in this aspect, affecting both the measuring accuracy, and the horizontal representation of the buildings (the latter affects both the scaling and the apparent positioning, cf. figure 3).

To minimize the scaling errors related to the grid resolution, each corner of the building (typically four) must be measured for mismatch. The mismatch of the building is represented as the mean mismatch of its corners. In order to simplify the data processing, the same number of corners are measured for all buildings.

Combining the results from individual corners does not equalize the positional error due to grid aliasing (see figure 3: these kinds of errors are actually part of what we try to describe (i.e. an accuracy measure that relates to the end product, with all its physical and stochastic imperfections).

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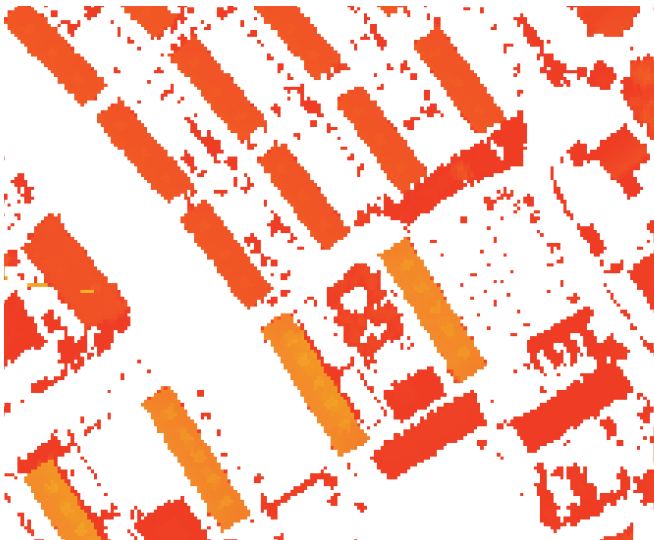
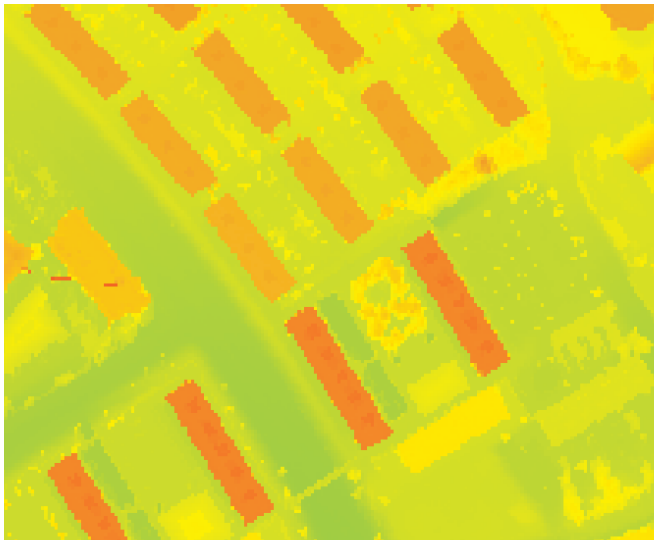


Figure 1: Top: DTM, center: DSM, bottom: normalized DSM (i.e. DSM-DTM), after threshold of 1.5 m has been applied

2.2 Visual estimation

A human doing a visual interpretation of an image is able to estimate the intersection of two well-defined lines, with an accuracy that is way better than one would expect from the reso-

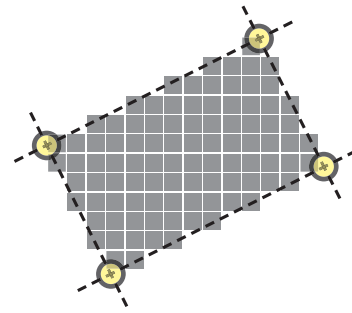


Figure 2: Lines representing the visual estimation of the sides to define the corners. A simple set square physically moved around on the screen is a useful guideline for the estimation. The points show where to measure the coordinates in the nDSM

Positioning and scaling issues due to the grid

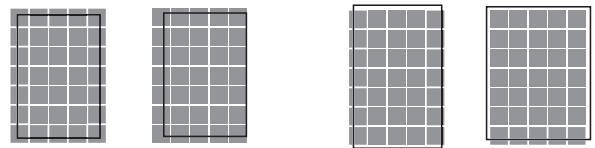
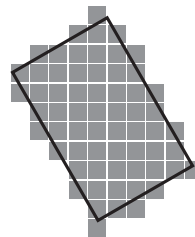
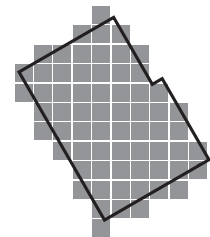


Figure 3: Leftmost two panels: The black boxes represent two buildings with same size, but positioned differently (and hence being registered with differing horizontal accuracy). Rightmost two panels: the black boxes represent two different buildings, each having the same size in the grid

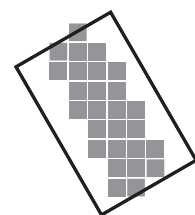
Suitable building



Invisible building feature



Offset cuts of the building



Time difference of aquisition

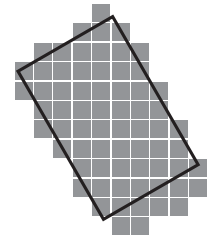


Figure 4: Four examples on situations where interpretation is needed. Imagine the pixels shown are in a 2 meter grid and represent buildings. The black lines represent the building in the reference data

lution and signal-to-noise ratio of the image.

This phenomenon has been utilized for many years in operational photogrammetry, where the rule of thumb says that an experienced stereo operator can determine the position of a well-defined point with a precision that is 3–5 times better than the image resolution (cf. e.g. Geoforum Danmark, 2005). In recent years, the phenomenon has been put on more firm

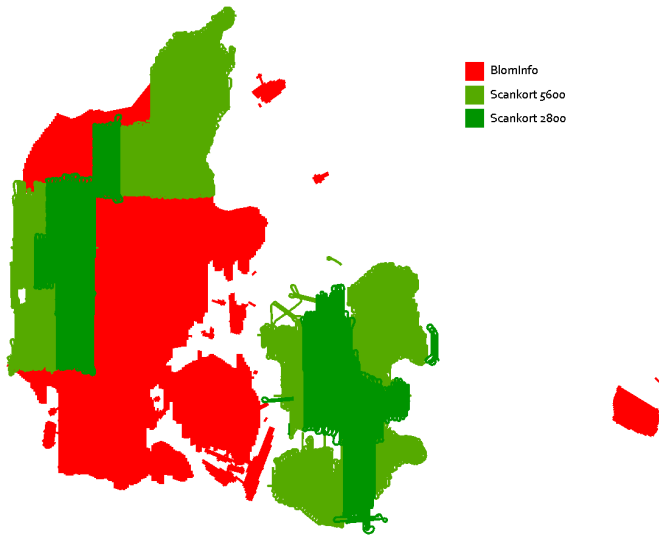


Figure 6: The coverage of the two companies, providing the LiDAR data

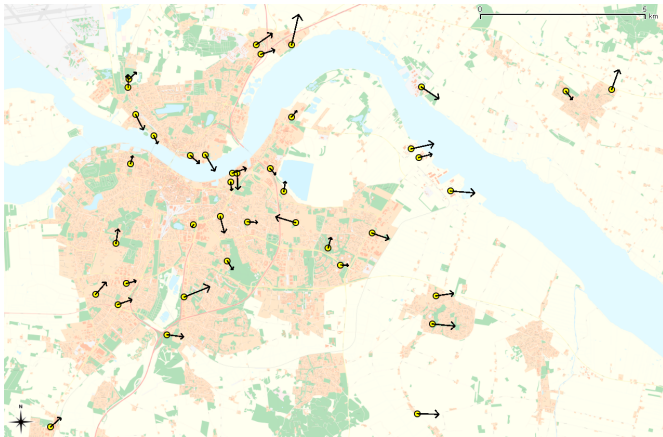


Figure 7: The testsite of the city Aalborg, with the resulting errors shown as arrows with direction and error size

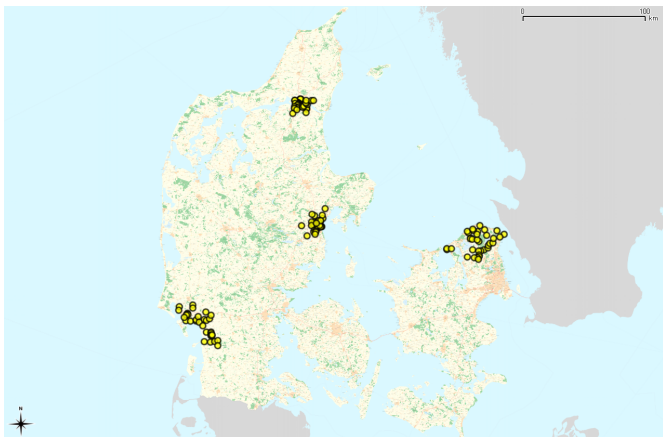


Figure 5: The four representative areas the analysis is based on

ground through the theory of compressive sampling (Candès, 2006; Candès and Wakin, 2008).

Here, we will not investigate the theoretical or practical background of the visual estimation problem any further. But in figures 1–2 illustrates the plausibility of high precision visual intersection estimation.

2.3 Implementation

A script using Python and GDAL (Warmerdam, 2010) was written to output the nDSM and converting it from ESRI ascii to GeoTIFF format.

For the measurements, QuantumGIS (QGIS, 2010) with the GeoReferencer plugin was used. The visual estimation was guided using a set square (or *drafting triangle*) slid directly on the LCD panel to align optimally along the building sides. This software is highly recommended, as it gives a simple and efficient workflow.

For the statistics and other output files, an Octave script was written.

3 Case study, DK-DEM

The Danish national DEM (DK-DEM) is the study object, and prime inspiration, for this work. The DK-DEM consists of a DSM and a DTM, both in the form of raster grids with a node distance of 1.6 m.

In the tender for the DEM, the specification for horizontal accuracy was set to 1 m. While this accuracy was most probably reached for the underlying point cloud, it was never checked whether further inaccuracies had crept in during the post processing, gridding, georeferencing, etc. phases.

In order to check for these kinds of mistakes, the algorithm described above, was put to use in the case of DK-DEM

To keep the scale of the analysis small, it was decided to use national topographic maps as reference data, and to limit the measurements to four representative areas (see figure 5).

DK-DEM is based on LiDAR data provided by two different companies, using three different scanners. This has been taken into account when selecting test sites (compare figures 5 and 6).

After initial implementation and testing of the method, the work on the DK-DEM took approximately 40 hours, including nDSM creation and load times. The biggest challenge (and most time consuming part of the process) was to identify a sufficient number of suitable buildings. The actual registration and data processing went quite fast.

3.1 Results

All in all, 568 building corners were measured for 142 buildings

For each test site an output file is generated, containing direction and size of the deviations for each building, see figure 7 for an example.

The results do not take into account the accuracy of the topographic maps and of the the visual estimation. So one could argue that this makes the result relative. But since the production of DK-DEM and the topographic maps are largely independent, when one arrives at largely identical results, we can apply a rule of thumb that the absolute error is no larger the relative deviation. A more formal argumentation (which will bring us to the same conclusion) is outlined in appendix A, below.

The details of the accuracy analysis are described in Hawa (2010a,b,c,d).

- The calculated RMS for the horizontal error for the four test sites, spans from 55 cm to 82 cm.

- The overall coordinate difference in the northing direction is close to 0: by far the largest part of the difference is in the easting direction.
- No systematic differences appear to exist between results based on data from the two different providers.
- No systematic differences appear to exist between results based on data from the two different reference data sets.

It is notable that the model appears to be shifted mostly towards the east (compared to the reference data). The reason for this is unknown – and as we work at small scales compared to the grid node length, a deeper understanding will require a very thorough analysis (which is way outside the scope of this work).

4 Conclusion

The method described can be used to determine the horizontal accuracy of a raster DSM and corresponding DTM. It is a simple method where no advanced software is needed.

The procedure can be executed quickly, and hence offers an excellent price/performance ratio.

The method is mainly a manual process, where coordinate differences are measured between reference objects in the DEM and suitable reference data. In the case study, two different topographic map data sets were used.

For the case-study of DK-DEM an overall horizontal accuracy on the order of 1 m was expected and observed.

Acknowledgements: This work was carried out during an internship at the National Survey and Cadastre (KMS), as part of the author’s studies for the degree of cand. geom. (surveying) at Aalborg University.

My colleagues at the KMS have influenced the work significantly: the algorithm described was originally proposed by Simon Lyngby Kokkendorff, Thomas Knudsen, Brian Pilemann Olsen, and Brigitte Christine Rosenkranz.

Brian Pilemann Olsen and Brigitte Christine Rosenkranz were instrumental in gathering the data needed.

Simon Lyngby Kokkendorff wrote the Python script used.

Thomas Knudsen guided in the use of QGIS plugins and in the selection of background literature. He also read and commented on numerous versions of this paper.

Lolita Bahl, manager of the Reference Networks Division at KMS, gave me the opportunity to work on this interesting subject.

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Emmanuel J. Candès and Michael Wakin. An introduction to compressive sampling. *IEEE Signal Processing Magazine*, 25(2):21–30, March 2008. URL <http://dsp.rice.edu/files/cs/CSintro.pdf>.

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M. Nour Hawa. Test site Århus. In *this volume*, page 18, 2010b.

M. Nour Hawa. Test site Esbjerg. In *this volume*, page 17, 2010c.

M. Nour Hawa. Test site Nordsjælland. In *this volume*, page 19, 2010d.

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A The variance of the difference between two elements is greater than the variance of the elements (proof)

We take as our starting point the often used assumption that the absolute error of point coordinates in a collection of geo-data (e.g. the difference between the point coordinates and the coordinates of the corresponding “true physical” points) are samples of a random distribution with some standard deviation, which reflects the quality of the dataset.

Under this assumption the relative error between two different, *uncorrelated* datasets, which describe the same physical points, will be randomly distributed with a standard deviation which is *greater than the individual standard deviations* – simply by the basic addition formula for variances:

Given two datasets A and B , let C_A and C_B denote the point coordinates (say, the easting coordinates) and E_A and E_B denote the absolute errors of these coordinates: e.g. $E_A = C_A - C_{truth}$.

Then we have for the *relative error* between the coordinates in the two datasets:

$$\begin{aligned} \text{Var}(C_A - C_B) &= \text{Var}(C_A - C_{truth} - C_B + C_{truth}) = \\ \text{Var}(E_A - E_B) &= \text{Var}(E_A) + \text{Var}(E_B) - 2\text{Cov}(E_A, E_B) \Rightarrow \\ \text{Var}(E_A - E_B) &\approx \text{Var}(E_A) + \text{Var}(E_B) \end{aligned}$$

Hence, *the variance of the difference is larger than the largest variance of the individual elements*.

If the individual absolute errors are normally distributed and the errors are independent (as assumed), then the relative error between the two datasets will also be normally distributed, (cf. e.g. http://en.wikipedia.org/wiki/Normal_distribution for a brief outline).

B Diagrammatic outline of the procedure

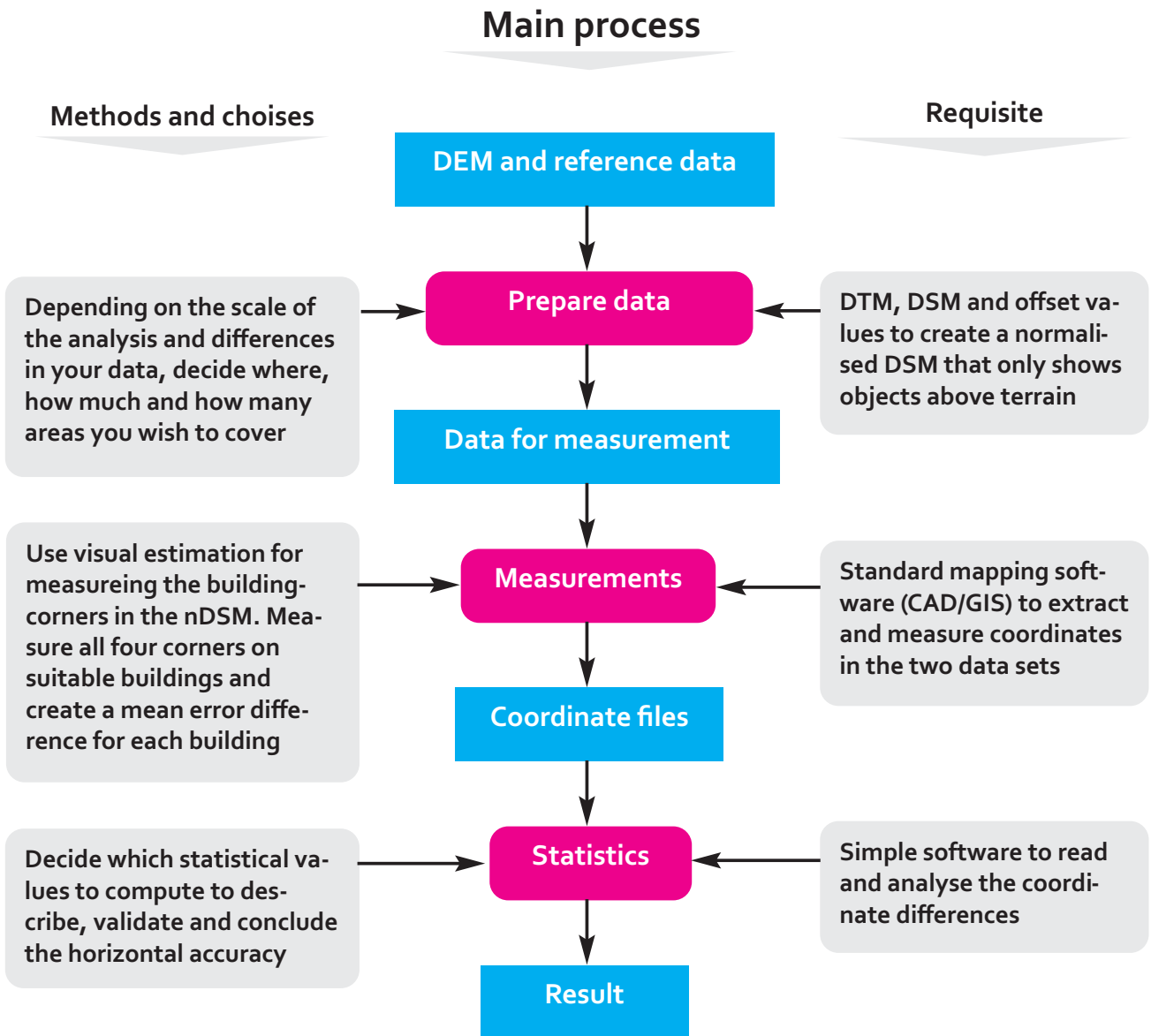


Diagram Legend

Process

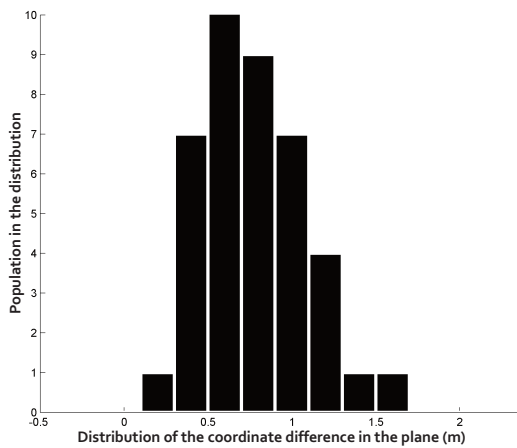
Input/output

Test site reports

In the following four pages, we reprint the test site reports for each of the four test sites in the DK-DEM horizontal accuracy project.

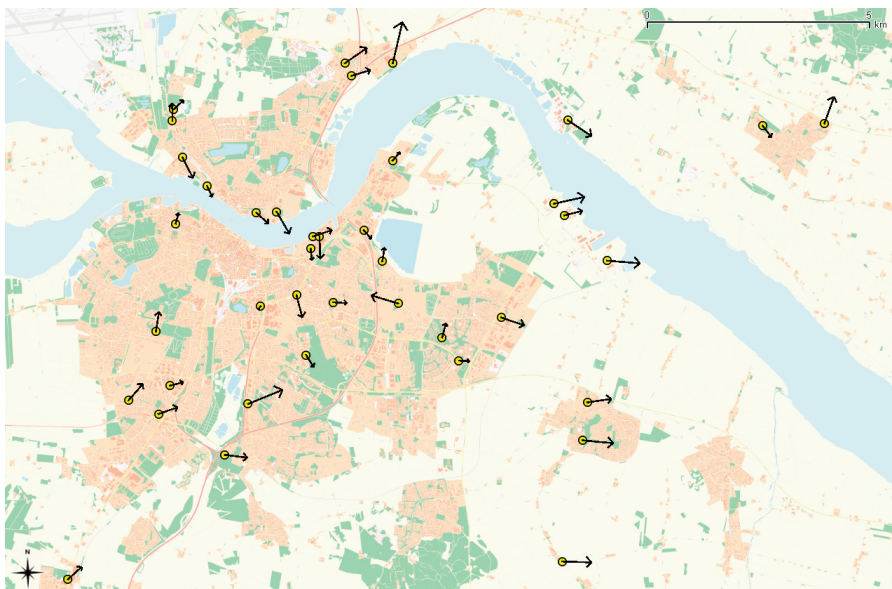
Test site - Aalborg

| | |
|-------------------------------------|---|
| Reference data | TOP10DK (Danish topographic map) |
| Number of measured reference points | 160 |
| Object type | Well-defined building corners on large buildings, non-parallel with the the north/south or east/west axis |
| Scanner and operator | Riegl Litemapper 5600 / Scankort |
| Algorithm | 4 measurements per building, combined into one value |



Results

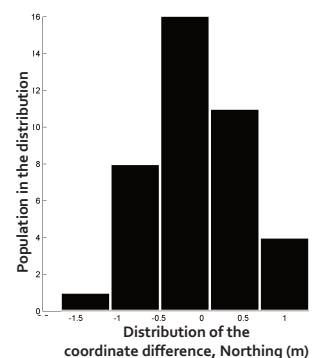
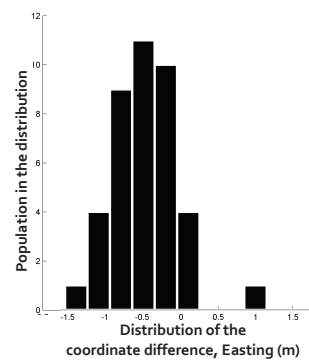
| | |
|------------------------------|--------|
| RMSE, plane | 82 cm |
| Mean, plane | 76 cm |
| Maximum deviation | 156 cm |
| Easting, mean error | -47 cm |
| Easting, standard deviation | 44 cm |
| Northing, mean error | -7 cm |
| Northing, standard deviation | 52 cm |



The yellow points represent the measured buildings around the Århus area.

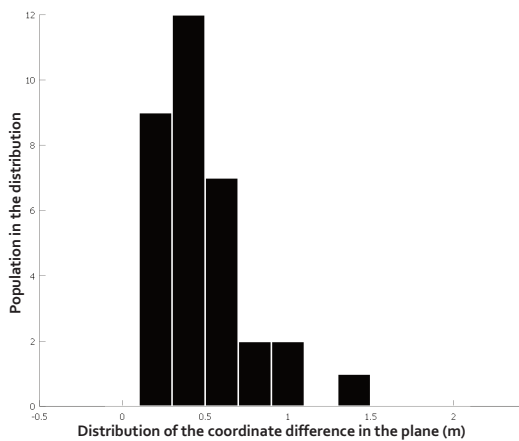
The arrows show the directions and RMSE value, of the coordinate differences. Arrows are exaggerated by a factor 500.

As also reflected by the result table, the DEM appears to be shifted approximately 50 cm eastwards.



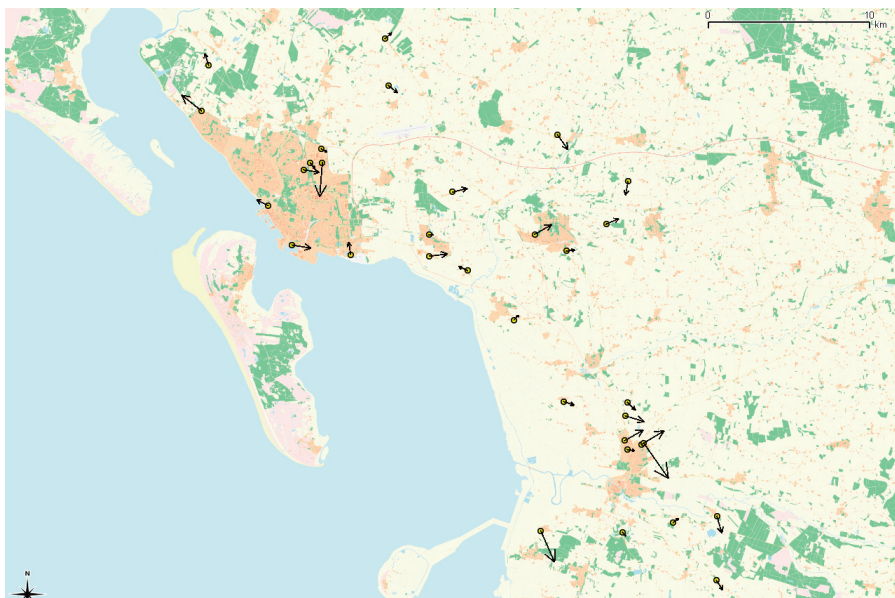
Test site - Esbjerg

| | |
|-------------------------------------|---|
| Reference data | FOT (Danish topographic map) |
| Number of measured reference points | 132 |
| Object type | Well-defined building corners on large buildings, non-parallel with the the north/south or east/west axis |
| Scanner and operator | Optech / BlomInfo |
| Algorithm | 4 measurements per building, combined into one value |



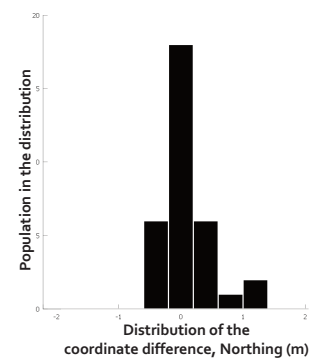
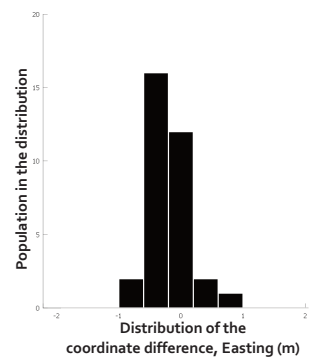
Results

| | |
|------------------------------|--------|
| RMSE, plane | 55 cm |
| Mean, plane | 48 cm |
| Maximum deviation | 134 cm |
| Easting, mean error | -23 cm |
| Easting, standard deviation | 31 cm |
| Northing, mean error | 8 cm |
| Northing, standard deviation | 40 cm |



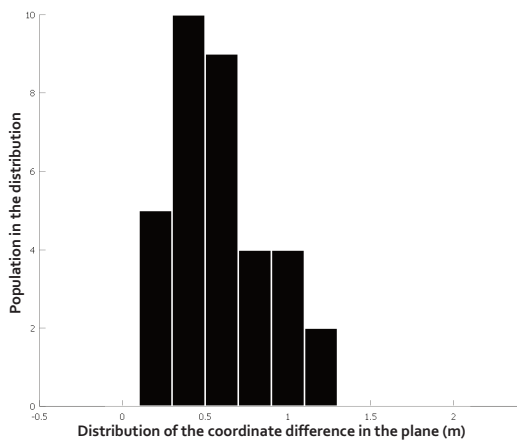
The yellow points represent the measured buildings around the Esbjerg area.

The arrows show the directions and RMSE value, of the coordinate differences. Arrows are exaggerated by a factor 2000.



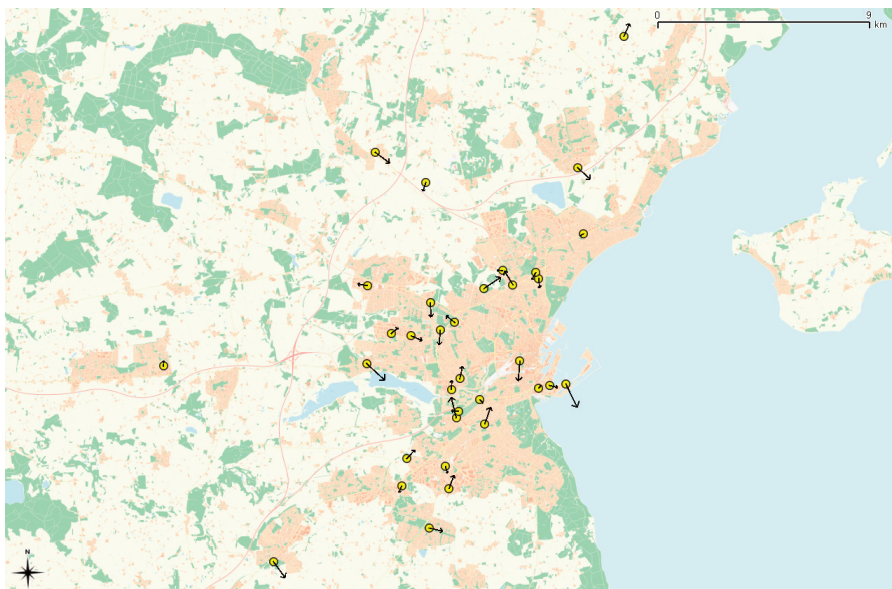
Test site - Århus

| | |
|-------------------------------------|---|
| Reference data | TOP10DK (Danish topographic map) |
| Number of measured reference points | 136 |
| Object type | Well-defined building corners on large buildings, non-parallel with the the north/south or east/west axis |
| Scanner and operator | Optech / BlomInfo |
| Algorithm | 4 measurements per building, combined into one value |



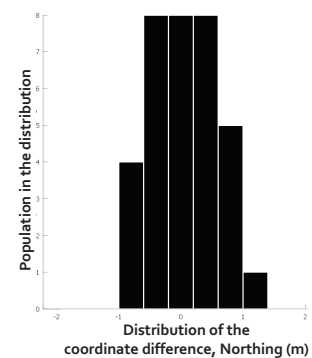
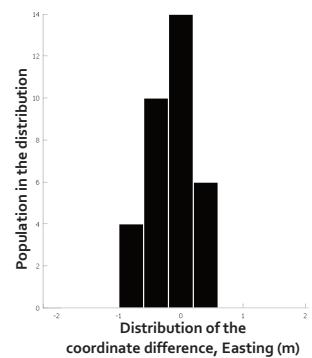
Results

| | |
|------------------------------|--------|
| RMSE, plane | 64 cm |
| Mean, plane | 58 cm |
| Maximum deviation | 122 cm |
| Easting, mean error | -14 cm |
| Easting, standard deviation | 38 cm |
| Northing, mean error | 6 cm |
| Northing, standard deviation | 53 cm |



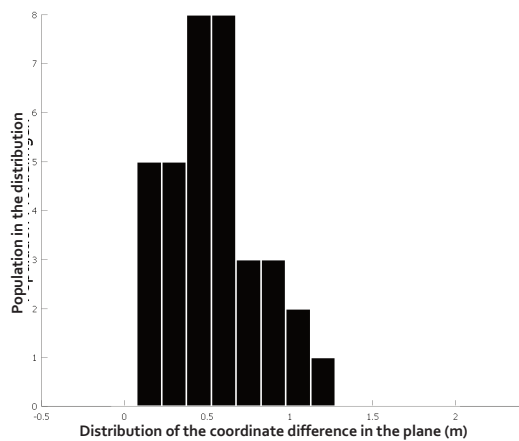
The yellow points represent the measured buildings around the Århus area.

The arrows show the directions and RMSE value, of the coordinate differences. Arrows are exaggerated by a factor 900.



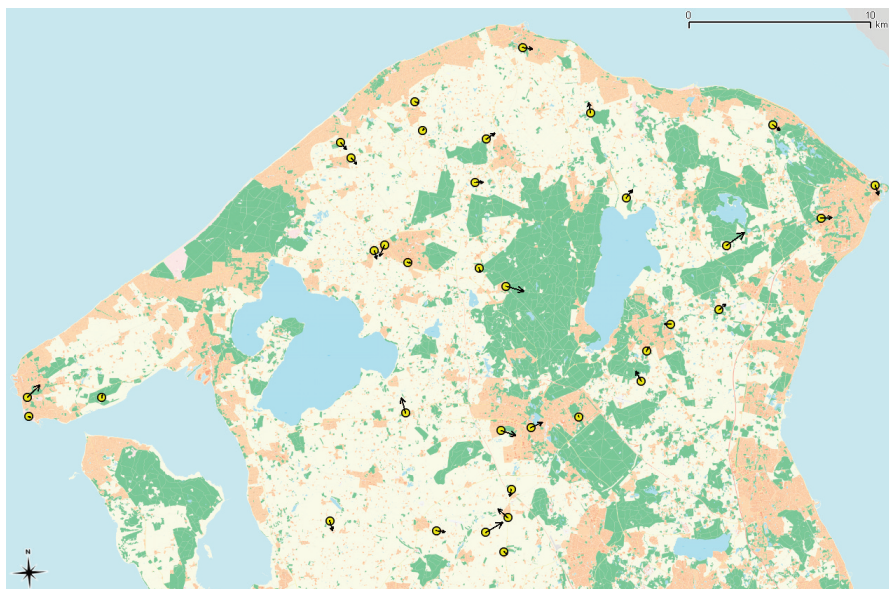
Test site - Nordsjælland

| | |
|-------------------------------------|---|
| Reference data | FOT (Danish topographic map) |
| Number of measured reference points | 140 |
| Object type | Well-defined building corners on large buildings, non-parallel with the the north/south or east/west axis |
| Scanner and operator | Riegl Litemapper 5600 / Scankort |
| Algorithm | 4 measurements per building, combined into one value |



Results

| | |
|------------------------------|--------|
| RMSE, plane | 60 cm |
| Mean, plane | 53 cm |
| Maximum deviation | 123 cm |
| Easting, mean error | -25 cm |
| Easting, standard deviation | 38 cm |
| Northing, mean error | -4 cm |
| Northing, standard deviation | 40 cm |



The yellow points represent the measured buildings around the Nordsjælland area.

The arrows show the directions and RMSE value, of the coordinate differences. Arrows are exaggerated by a factor 500.

