TECHNICAL REPORT NO. 1

Measurement and Computation of the new Oeresund Loop

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DANISH MINISTRY OF THE ENVIRONMENT

National Survey and Cadastre

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National Survey and Cadastre, technical report series no. 1
ISBN 87-92107-09-5
Technical Report
Published 2008-06
This report is available from www.kms.dk

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Technical report handed over to the NKG Working Group for Height determination at the meeting in Akranes, Island, June 13-14, 2005

Abstract. *In order to connect the height systems of Denmark and Sweden the Sound has been crossed repeatedly in the past at its northernmost end. From this, two small local loops could be set up from optical and hydrostatic water crossings. However, due to the recent completion of the Oeresund Link a new crossing could be established at the southernmost end of the Sound, giving for the first time the possibility to close a loop around the Sound. The crossing has been accomplished by motorized leveling in the year 2000 as a joint enterprise by National Land Survey of Sweden (NLS) and National Survey and Cadastre of Denmark (KMS).*

The setup of the new loop is carefully defined and a description of all measurements involved is given. The measuring quality of the new crossing is shortly analyzed and compared to the Danish Third Precise Leveling. Based on the closing errors of the new loop and a local loop, which can be set up from an optical crossing and a hydrostatic leveling as well, conclusions are drawn on the reliability of the crossings involved and their usefulness for connection of the national height systems.

For the sake of consistency gravity has been applied to the leveling, however, for practical purposes the effect is almost insignificant. This also goes for postglacial land uplift corrections.

Acknowledgement: *Thanks to our Swedish colleagues from the NLS for providing the Swedish leveling data, in particular to Per-Ola Erikkson always available to answer new questions. Thanks also to J. Mäkinen from the Finnish Geodetic Institute (FGI), delivering the uplift grid, K. Johansen (KMS), setting up the on-shore leveling line from the Danish Third Precise Leveling, and G. Strykowski (KMS) for discussing gravity reduction using the GeoGrid program.*

1. Introduction

1.1 Historical background. In order to connect the height networks of Sweden and Denmark, i.e. the connection of the continental European networks with the networks of the Nordic countries, the Sound has to be crossed. Nowadays, this possibly could be achieved with sufficient accuracy by GPS, formerly, however, other solutions had to be found. Thus, the Sound was crossed by two optical water crossings and a hydrostatic leveling all carried out at the narrowest place (5km) at the northernmost end, Helsingør/ Hälsingborg. (For the sake of completeness three additional optical crossings from 1896/1898 and 1981 should be mentioned, se the references below. However, all of them are of doubtful quality.)

The first optical water crossing connecting the national First Precise Leveling networks took place as early as in 1896/1898, cf. the crossing K-T in Den Danske Gradmaaling (1909). As such crossings can be highly affected by deflections of the vertical as well as asymmetrical refraction the national Second Precise Leveling networks were connected by a hydrostatic leveling in 1939, cf. Nørlund (1946). Indeed, comparing these crossings a major deviation of 17mm turned out, ibid. p.81. Of course, this could be caused by poor quality of the optical crossing, but on the other hand the Sorgenfrei/Tornquist fault zone, separating the Fennoscandian Shield from the Norwegian Danish Basin, is crossing the Sound just in the area considered, cf. figure 1. Thus, irregular vertical movements of the reference benchmarks, occurred in the intervening time span of about 40 years, could not be excluded. Besides, due to the postglacial Fennoscandian land uplift, cf. Ekman (1996), there are no stable points in the Nordic countries. Nevertheless, if this is the only cause of a point's vertical movement we shall use the term "stable" throughout this report.

In order to investigate this further a new optical crossing, recorded in Bedsted Andersen et al. (1986), was established in 1980, by the Danish Geodetic Institute (G.I.) and the Swedish NLS. As decided by the Leveling Working Group of the Nordic Geodetic Commission (NKG) the new crossing was designed as a repetition of the old one performed under the same conditions, as far as possible. Supposing this would imply the same systematic errors the difference of both crossings, based on a time span of about 100 years, could give an indication of the stability across the Sound. This concept possibly has been successful, because recomputing the old crossing and comparing it with the new one from 1980 a deviation of just 2mm was found, cf. Bedsted Andersen et al. (1986), p.35. As this result is subject to an estimated mean error of 5mm it was concluded "that no detectable relative height movements have taken place across the Sound from 1896 up to now (1980)". However, the deviation is based on the somewhat doubtful leveling from 1984 in table 6; using the value from 1980/81 instead the difference would be 7mm.

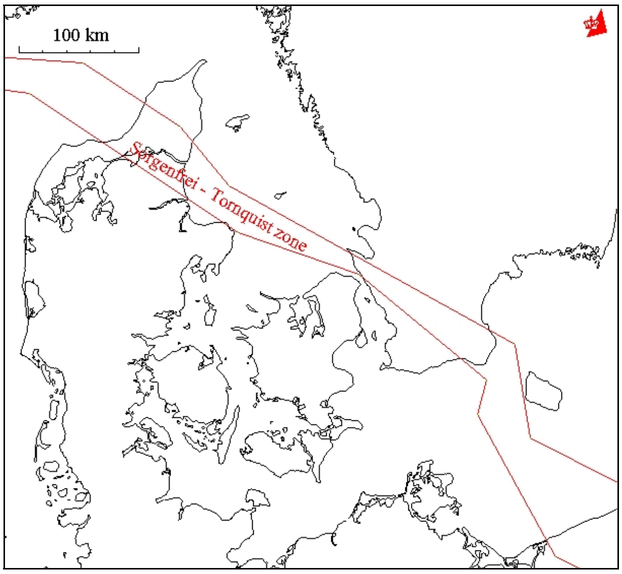


figure 1: Location of the Sorgenfrei-Tornquist zone in the area of the Sound, from Abramovitz, T. et al. (1997)

1.2 Purpose and planning of the new measurements. In the year 2000 a new possibility to cross the Sound came into existence due to the completion of the Oeresund Link (16km) connecting the cities of Copenhagen and Malmö, cf. sect. 1.3 for constructional details. Based on a proposal of the Height Determination Group of the NKG in summer 1999, a common decision was made by NLS and KMS to use the Link for the connection of the new national Third Precise Levelings conducted during the last two decades. If the height determination along the Link could be done with sufficient accuracy the height connection between Sweden and Denmark would be strengthened. Moreover, connecting the Link to the new leveling lines along the Sound and using the optical crossing from 1980 it would be possible for the first time to close a loop around the Sound, cf. figure 2, in order to check measuring quality. Hence, the primary goal of this report is the computation of the closing error taking into account postglacial uplift of the benchmarks occurred within the measuring time span of the loop. Moreover, we shall compute the closing error of the local loop at Helsingør/Hälsingborg, which can be set up from the optical crossing (1980) and the hydrostatic leveling.

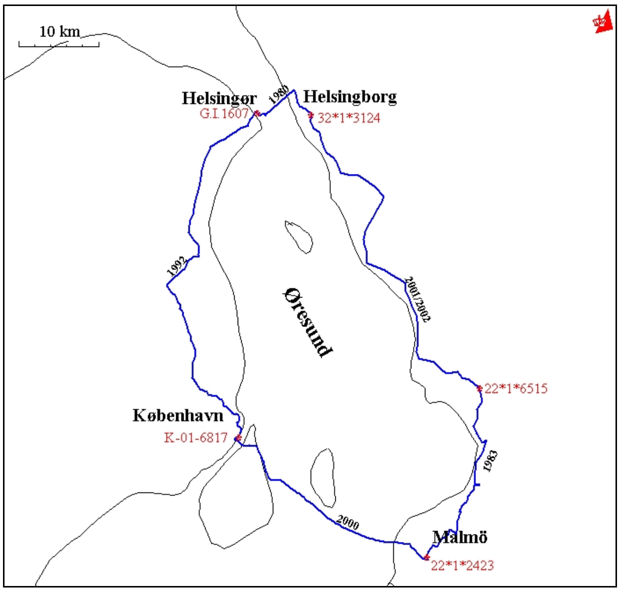


figure 2: The Oeresund Loop and its points for temporal connection

As the networks to be connected are coming from motorized geometric leveling it was clear from the beginning that the height determination of the Link should be conducted in the same way. For control purposes the measurements on the high bridge should be supplemented by motorized trigonometric leveling with sight lengths up to 500m to avoid all the intermediate setups needed for geometric leveling. However, due to the open sea strong wind is frequently occurring in the area. Also, heavy machines for road construction would be working during the measurements. Thus, it was doubtful if the leveling could be done with sufficient accuracy within the short period of time granted by the Oeresundsbro Konsortiet. Nevertheless, the completion of the Link was a one-time opportunity to strengthen the height connection across the Sound, therefore the measurements were started in the beginning of April, 2000.

1.3 The construction of the Link. According to information from the Internet, www.oeresundsbron.com, the Link is consisting of a 4 km long tunnel, an artificial island, Peberholm (4 km), and an 8 km long bridge made up by the western approach bridge (3 km), the cable-stayed high bridge (1km) with a free span of 490 m, and the eastern approach bridge (4 km), cf. figure 3. The tunnel is necessary for security reasons due to the airport of Kastrup nearby.

figure 3: High Bridge and approach bridges of the Oeresund Link

Inside the tunnel (width 40m, height 7m, lowest point -20m) a 4-track motorway and a double track railway are running side by side through 4 tubes immersed in a ditch at the bottom of the sea and covered by stones. The soil from the immersion of the tunnel has been reused to build up the island of Peberholm, surrounded by shallow water and measuring about 300m across. The island is needed for the transition of the four tunnel tracks to the two levels of the bridges with the motorway on the upper deck about 10m above the railroad level. Motorway and railroad deck are connected through a stiff steel framework. The approach bridges are supported by piers, whereas the high bridge (max. height 70m) is tied to two pair of pylon legs, 204m high.

2. The measurements

2.1 On-shore levelings. Except for a short line of precise geometric foot leveling, connecting the optical crossing from 1980 and the Danish leveling network of that time at Helsingør, the on-shore lines of the new loop are part of the national Third Precise Leveling networks. Both networks have been measured by geometric motorized leveling, cf. Becker (1987), i.e. rods and level are moved by vehicles specially designed for transportation, and levels of the type Zeiss NI 002 as well as calibrated invar rods are applied in accordance with the Nordic guidelines for leveling, cf. Becker and Bedsted Andersen (1986). As regards precision motorized leveling is comparable to foot leveling, i.e. a mean error of about 1mm of 1km double run as found from the adjustment of the Danish Third Precise Leveling, cf. Schmidt (2000). However, productivity, essential for the measurement of the Link, is much higher, say, 2 km single run per hour.

In precise leveling the proper observation of a section height difference is a double run, i.e. the mean of one measurement (single run) in either direction. On location the discrepancy ρ between forward and backward run is tested. If the absolute value of $\rho' = \rho / \sqrt{L}$ is exceeding a certain limit k both runs are rejected and a new double run has to be measured. Here ρ is in mm, L is the section length in km, and k=1.8 or k=2.0 for Danish or Swedish leveling.

2.2 The Link. The leveling of the Link has been accomplished by one team from either country, using the same procedures and equipment as for the national Third Precise Levelings. The work took place within 8 days in chilly weather (3-8 C˚). Even though the leveling was planned from day to day in accordance with the weather conditions some of the measurements had to be conducted under difficult circumstances, e.g. strong wind in plain sunshine.

Wherever permitted by the Oeresundsbro Konsortiet proper leveling bolts were established by the national road authorities. Otherwise, removable bolts in the tunnel or tops of solid vertical screws were used for leveling control. Along the approach bridges the control points were pointed out on every second pier corresponding to leveling sections of 480 m length. Stable leveling points kept in the database of KMS are indicated by bold types in table A4. Of course, the piers were used for support of the level and the rods during the leveling. As a rule, neighboring leveling points were connected by 5 or 6 double runs.

Trigonometric leveling of the high bridge was carried out by an extra Danish team. The results were in good agreement with the geometric leveling, thus it was concluded that the latter was influenced by vertical movements much less than expected. For different reasons the trigonometric results have not been used in our computations.

Finally, as recorded by R. Forsberg (KMS) gravity was measured at 6 new points, cf. app.2, sect.2.1, in order to improve gravity interpolation along the Link, which is necessary for a consistent leveling adjustment.

figure 3.1: Motorized leveling on the High Bridge

2.3 The optical water crossing (1980). The crossing performed during 2 weeks in the autumn of 1980 is connecting the top of the Telegraph Tower of the Castle of Kronborg with the top of the cliff of Pålsjø, a few kilometers north from Hälsingborg, cf. figure 4. These sites have been used, too, for the first optical crossing. They are located at a maximum height (30 m) above sea level in order to lessen asymmetrical refraction at the observation sites.

figure 4: Special locations in the area of Helsingør/Helsingborg

The crossing has been accomplished applying the so-called fjord crossing method often used in Norway, i.e. the height angles of a pair of special target plates, attached to a leveling rod raised close to the opposite site, are measured simultaneously from both sites by means of the scale of the tilting screw of a Wild N3 level, cf. figure 5. From this, the height difference of the sites' reference benchmarks can be computed twice. As can be shown each of these values, which are affected by the earth's curvature, the level's collimation error, and refraction, can be interpreted as an ordinary leveled setup height differences (backward - forward reading) with leveling rods fictitiously raised at the reference benchmarks, however, the sight lines are of very different lengths. Further details on measurements and computations can be found in Bedsted Andersen et al. (1986).

figure 5: Optical water crossing from the top of Telegraph Tower aiming to the signal plates in Sweden

It turned out from the computations that the height angles were highly affected by unsymmetrical refraction. Consequently, all readings taken under unfavorable meteorological conditions, i.e. about half of the entire number, were excluded from the final computation.

Last but not least, the benchmark on top of the Telegraph Tower had to be connected to the existing leveling network by measuring its height above the benchmark at the bottom of the tower. This was achieved through a vertical steel tape connecting both points. The length of the tape corresponding to a given traction and temperature was found from interferometer measurements.

3. Lining up the Oeresund Loop

3.1 The points for temporal connection. In order to set up the loop leveling lines from different decades had to be connected. Wherever these levelings are overlapping any point of the overlap could be chosen for connection, however, the vertical movement of the connection point should be estimable as good as possible, cf. sect.5. With this in mind the connection points, shown in fig.2, have been carefully selected.

On the Danish side points preferably joining a group of subsoil benchmarks should be used for connection purposes. Frequently, the benchmarks of such group are standing in a row within a short distance, say, 50m. They always are leveled jointly to monitor vertical stability. In general, a subsoil benchmark is consisting of a well-founded granite pillar or a concrete block, the top of which is up to 1½m beneath the ground. Its proper motion (relative to the surrounding soil) is expected to be insignificant. Also, movements, which are nonlinear in time, are unlikely. Thus, subsoil benchmarks are considered as the most stable kind of leveling marker in Denmark.

Due to the lack of measurements there is no subsoil point in the area of Copenhagen to connect the Link with the Danish Third Precise Leveling. Anyhow, there are three different bolts, which could be used. Among them we have selected the bolt K -01-06817 located at the old, massive building of the Castle of Christiansborg (The Danish Houses of Parliament). According to preliminary results from the computation of land uplift in Denmark based on the three Danish Precise Levelings, the bolt seems stable relative to the nearest subsoil benchmark G.M.1393. However, both points could be on unstable ground, because they are close to the sea in a few meters height. Nevertheless, the bolt also seems stable relative to the remote subsoil group G.I.1803-1806 located inland close to the former Geodetic Institute’s laboratory.

In the Helsingør region the connection of the Third Precise Leveling and the optical crossing (1980) is complicated by inconsistent leveling results as recognized already in Bedsted Andersen et al. (1986), sect.II, 1.2.2. Investigating the levelings from 1940 on, cf. app.2, sect.1, the connection point finally chosen is the benchmark G.I.1607 of the subsoil group Hamlets Grav, se fig. 4.

In addition, several lines from the Swedish Third precise leveling had to be joint in order to connect the Helsingborg area with the Link. The points of connection, 032*1*3124, 022*1*6515, and 022*1*2423, have been pointed out by our Swedish colleagues claiming stability of these points in the last few decades.

3.2 Debugging the Link. It was obvious from the data delivered by NLS that the rejection limit of precise leveling, cf. sect. 2.1, had not been applied to the Swedish double runs of the Link. Catching up on this we noticed a rather large number of severe rejections on the eastern approach bridge. It could be shown from a closer investigation that this was caused by 2 leveling points, 4078/500 501 and 4078/500 502, both alternately moving within a few millimeters between 2 vertical positions depending on the day of observation. According to the leveling teams, this probably can be explained through heavy machines for road construction working close to these points on some of the days. In our computations we have used additional point numbers, i.e. the original ones extended by .1, to distinguish between both positions. However, due to some rejections the point 4078/500 502.1 became the end point of an antenna, therefore it will not occur in this documentation. It should be mentioned that point numbers of the type 4078/…are referring to the numbering system adopted by the national road authorities,

3.3 Configuration. Apart from the optical crossing the loop has been set up entirely by double runs obtained from single runs in the national data files, the latter corrected for scale graduation errors and temperature. All the double runs included have been accepted according to sect. 2.1 except for a few runs outside of the Link. Repeated double runs have been included separately, antennas have been removed. Along the Link the loop is consisting of line-shaped networks connecting the different sections of the Link, such that neighboring networks have just 1 point in common. Outside of the Link the loop is consisting of consecutive double runs without crossovers and just a few repetitions.

In total, the loop has been set up from 315 points connected by 502 double runs, in addition to the optical crossing from 1980 and the corresponding steel tape measurement. Double runs are listed at the end of this report. The circumference of the loop is about 192 km, but due to the large number of repeated double runs along the Link the accumulated length of all double runs involved is about 254 km (76 km along the Link).

A schematic outline in clockwise order is given in table 1. Bold point numbers are indicating the connection points in fig. 2. In case of leveling ΣL is denoting the accumulated length in km of the double runs included. The $\Delta\hat{H}$-values below are adjusted Helmert height differences according to sect. 4.2, except for the water crossing, which is orthometrically corrected, whereas the corresponding steel tape measurement is uncorrected. The sources are referring to the Swedish Third Precise Leveling data, the KMS leveling data files #DK_niDniv and #DK_niDprs, the G.I. field book GA.I, vol. 102, and Bedsted Andersen et al. (1986).

location	point no.	$\Delta\hat{H}$	method	ΣL	year	data source
Hamlets Grav	**G.I.1607**					
		-13.65976m	geom..foot lev.	2.8	1980	#DK_niDniv & GA.I, bind 102, p.128916, 128918
Telegr. tower, foot	K –06-09008					
		21.7468	steel tape	0.022	do.	GA.I, bind 102, p.129142
Telegr. tower, top	K –06-09170					
		-0.59106	opt. water cr.	4.8	do.	Bedsted A., p.30
Pålsjø Cliff	032*2*3116					
		2.23258	geom..mot. lev.	3.1	1980/81	Sw. 3rd prec. lev.
Kärn., found.wall	**032*1*3124**					
		-15.44517	do.	58.4	2001/02	do.
	022*1*6515					
		-1.33355	do.	32.2	1983	do.
	022*1*2423					
		3.48312	do.	6.4	2000	do.
Link, start	4078/500 503					
		-21.28167	do.	75.8	do.	#DK_niDniv & Sw. 3rd prec.lev.
Link, end	4078/100 501					
		5.52243	do.	14.5	1998/2000	#DK_niDniv
Christiansborg	**K –01-06817**					
		19.31278	do.	60.5	1992	#DK_niDprs
Hamlets Grav	**G.I.1607**	------------		-------		
		-0.01350		258.5 km		

table 1: Set up of the Oeresund Loop

4. Analysis and adjustment

4.1 Double run discrepancies. In order to compare the leveling of the Link with the Danish Third Precise Leveling we have computed sectionwise the average (av) and standard deviation (std) of the normalized discrepancies ρ', cf. sect. 2.1, from all the double runs of the loop included in the Link. As mentioned before all these runs have been accepted. The results are shown in table 2, where the last row is indicating the percentages of rejected double runs based on the total number of runs performed along the Link (accepted or not).

location:	east bridge	high bridge	west bridge	Peberholm	tunnel
av(ρ') :	0.25 mm	0.28	0.49	-0.11	-0.11
std(ρ') :	0.77 mm	0.87	0.76	0.99	0.97
number :	59	15	59	32	64
reject. pct.:	13	21	9	14	2

table 2: Statistics of double run discrepancies

According to Schmidt (2000), sect.3223, the rejection percentage of the Danish Third Precise Leveling is below 5%. Hence, the values above are larger than usual, however, the indicated averages and standard deviations of ρ' are rather common. In particular, the standard deviations along the bridges are almost identical with the average value 0.75 mm from the Danish Third Precise Leveling, cf. sect. 5322, ibid. We thus conclude that the leveling of the Link after rejection is comparable with the Danish Third Precise Leveling as regards discrepancies from forward and backward run.

4.2 Adjustment. Based on the assumption that all double runs are stochastically independent the loop can be adjusted in a simple way. Regarding the Link least squares adjustment is applied separately to the different networks connecting the sections of the Link, cf. sect. 3.3. That means, keeping fixed à priori the height of a single point of the network, $\Delta H_{12}=H_2-H_1$ is the observation equation of the double run from point P_1 to P_2 of length L, and $\mathrm{Var}\Delta H_{12}=\sigma^2 L$ is the corresponding variance. Outside of the Link multiple double runs have been adjusted by their mean. Note, prior to the adjustments double runs have been converted into Helmert height differences by adding the orthometric correction, cf. app.1, sect. 2.1 and 2.3.

The results from the sectionwise adjustments are shown in table 3, where $\Delta\hat{H}$ is the adjusted Helmert height difference, m.e.($\Delta\hat{H}$) is the corresponding estimated mean error, s is denoting the estimated mean error of 1km double run, whereas n and m is the number of double runs and leveling points involved. The last row is indicating the adjusted Helmert height difference of the Link and the corresponding estimated mean error, that is the square root of the sum of the m.e.($\Delta\hat{H}$)-values squared. Note, the surprisingly low value, 1½mm, is caused by the large number of repeated double runs, cf. sect.2.2.

location	point no.	$\Delta\hat{H}$	m.e.($\Delta\hat{H}$)	s	n	m
	4078/500 503					
east bridge		47.59273m	0.77mm	0.77mm	59	19
	4078/430 261					
high bridge		0.05892	0.63	1.21	15	4
	4078/410 261					
west bridge		-51.50084	0.72	0.87	59	14
	4078/300 511					
Peberholm		-18.68245	0.63	0.72	32	8
	4078/300 501					
tunnel		1.24997	0.48	0.51	64	14
	4078/100 501	------------	--------			
		-21.28167	1.46			

table 3: Results from the sectionwise adjustments of the Link

According to Schmidt (2000), p.31, s-values in the range from 0.6 to 1.0 are expectable, thus the values above are quite common. As regards the rather large s-value of the high bridge point movements during the leveling could not be detected from the data, more likely, the value is caused by a single doubtful double run, which, however, had to be accepted according to sect.2.1.

The loop closing error has been computed by summing up the height differences in table 1. As is seen a value of -13.50 mm has been obtained. Also this is in good agreement with the Danish Third Precise Leveling. The mean error of the computed closing error can be sufficiently estimated from the dominating contribution from the on-shore leveling lines, the length of which is the length of the loop (192km) minus the length of the Link (16km). Assuming a mean error of 1 mm per 1 km double run the estimated mean error of the closing error is $\sqrt{192-16} \approx 13$ mm.

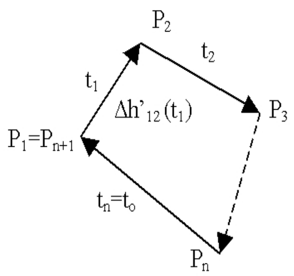
Finally, with reference to possible future remeasurements of the Link, we give the adjusted height differences $\Delta\hat{h}$ between the stable leveling points recorded in the files of KMS. The $\Delta\hat{h}$-values of table 4 are coming from ordinary adjustments of measured double runs, i.e. without applying orthometric corrections.

point no. 4078/:	500 503		400 506		400 502		300 511		300 502		300 501		100 502		100 501
$\Delta\hat{h}$:		49.40225m		0.02898		-53.28031		-16.26851		-2.41392		-1.55703		2.80697	
lev. distance :		4.3km		0.5		4.0		3.1		0.1		4.2		0.1	

table 4: Adjusted height differences of selected points of the Link

Still ignoring these corrections a loop closing error of -12.94 mm has been found, in addition to the adjusted height difference of the Link, $\Delta\hat{h} = -21.28157$ m. Comparing this with the results above it is obvious that the effect of the orthometric correction is insignificant from a practical point of view.

5. Loop closing errors and land uplift



The closing error in sect.4.2 has to be corrected for vertical movements of the connection points. In order to derive a correction formula consider the loop in the figure to the left composed of leveling lines from different years t_i connecting the points P_i, and let $\Delta H'_{i,i+1}(t_i)=\Delta H_{i,i+1}(t_i)+d\varepsilon$ denote the corresponding orthometric height differences, where $\Delta H_{i,i+1}(t_i)$ is the true value and $d\varepsilon$ the error induced from leveling. Assuming linearity of the movements given by annual uplift rates a_i, it holds

$$\Delta H_{i,i+1}(t_i) = \Delta H_{i,i+1}(t_o) + (a_{i+1} - a_i)(t_i - t_o)$$

where t_o is a certain reference year adopted à priori. Summing up along the loop the sum $\Sigma\Delta H_{i,i+1}(t_o)$ is vanishing, since orthometric heights are consistent, cf. app.1, sect. 2.1. Hence, let a_o be any constant value, then it can be easily shown that the closing error corrected for vertical movements can be written

$$\Sigma d\varepsilon = \Sigma\Delta H'_{i,i+1}(t_i) - \Sigma(a_i - a_o)(t_{i-1} - t_i)$$

Note the equation is not depending on the reference year t_o. Obviously, the larger the time span (t_{i-1}, t_i) the better an estimate of uplift is needed.

Annual uplift rates relative to mean sea level can be computed at specific points from repeated leveling and sea level observations. However, final values in the Oeresund region computed by the national surveying authorities are not available, yet. Nevertheless, established by Ekman (1996) there exists a continuous model of Fennoscandian postglacial uplift relative to mean sea level, which we have used in our computations. Other models possibly more refined are available, but no matter which model is chosen it cannot provide the actual uplift of specific points, if they are affected by local movements or proper motions. This is the reason why the connection points of the Oeresund Loop have been selected so carefully.

The uplift values applied have been interpolated from a numeric grid based on Ekman's map and handed over to the Height Determination Group by J. Mäkinen (FGI). They are corresponding to (a_i-a_o) in the formula above, where a_o is the uplift of the mean sea level. The values in mm/y are given in the table below.

Hamlets Grav	Kärnan, found. wall	Kärnan, tower chamber			Christiansborg
G.I.1607	032*1*3124	032*2*3109	022*1*6515	022*1*2423	K -01-06817
-0.13	-0.11	-0.11	-0.23	-0.34	-0.33

table 5: Annual uplift rates from Ekman

Using the measuring years from fig.2 we get from table 5, $\Sigma(a_i-a_o)(t_{i-1}-t_i)$=-0.25mm, i.e. the closing error in sect.4.2 is practically unchanged. Hence, we conclude the closing error corrected for postglacial land uplift is about -13mm, but recall, proper motions e.g. are not taken into account.

6. The loop set up from optical and hydrostatic water crossing

To line up the loop the crossings have to be connected by local leveling lines, among others the line from Pålsjø Cliff to Kärnan, cf. fig. 4, which has been measured repeatedly in the past. The results are given below. Note, the value from 1984 seems unreliable and is not used.

	1896/97	1939	1956	1980/81	1984
	9.217 m	9.21735	9.2176	9.21976	9.2247
source:	Bedsted A., p.21	Nørlund, p.25	NLS	Sw.. 3rd prec. lev.	Bedsted A., p.21

table 6: Height differences from Pålsjø Cliff to Kärnan, tower chamber

An outline of the loop is given in table 7, where the connection points are indicated by bold types. The $\Delta\hat{h}$-values are adjusted measured height differences without applying orthometric corrections. The circumference of the loop is about 17 km and a loop closing error of -2.13 mm has been found from summing up the height differences. From table 5 we get, $\Sigma(a_i-a_o)(t_{i-1}-t_i)$=0.80 mm, i.e. the closing error corrected for postglacial uplift is about -3mm.

location	point no.	$\Delta\hat{h}$	method	year	data source
Hamlets Grav	**G.I.1607**				
		-13.65975m	geom..foot lev.	1980	cf. table 1
Telegr. tower, foot	K –06-09008				
		21.7468	steel tape	do.	do.
Telegr. tower, top	K –06-09170				
		-0.5903	opt. water cr.	do.	do.
Pålsjø Cliff	032*2*3116				
		9.21976	geom..mot. lev.	1980/81	table 6
Kärn., tower	**032*2*3109**				
		-35.81085	geom. foot lev.	1939	Nørlund, p.24
	0-ref. point (S)				
		-0.01137	hydrostatic lev.	1939	Nørlund, p.79
	0-ref. point (DK)				
		1.05335	geom. foot lev.	1939	Nørlund, p.22
City Hall	K –06-09011				
		18.94986	do.	1940	Nørlund, p.23
Hamlets Grav	G.I.1606				
		-0.89963	do.	1940	G.I. comp.vol., p.2077 ff.
Hamlets Grav	**G.I.1607**	-----------			
		-0.00213			

table 7: Set up of the loop from optical and hydrostatical crossing

7. Gravity processing.

7.1 Preliminary remarks. According to theory leveling should be computed from geopotential or orthometric height differences (both are based on gravity), but not from the field measurements, cf. app1, sect. 2.1-2.3. However, it turned out that this did not really matter, cf. fig. 8.

As we know from the Danish Third Precise Leveling the deviations of leveled height differences from corresponding orthometric height differences are hardly summing up along a loop in a low and flat area with smooth gravity like the Oeresund region. Moreover, they normally are rather small, say, less than 0.1mm per km. However, gravity behaves irregular in the area of Helsingør/Hälsingborg, cf. fig.6. In addition, the leveling of the Link certainly is incomparable with normal leveling, since measurements have been performed not only on the ground but also on the bridges and in the tunnel.

Due to these considerations we have decided to apply the strict approach, i.e. leveling combined with gravity, even though this makes the computations a good deal more cumbersome. In the present case we have used orthometric height differences, because the orthometric corrections immediately are illustrating the effect of gravity on leveling.

7.2 Particular circumstances. Usual textbooks on leveling are referring to leveling on the ground, thus the common computation formulas of Bouguer anomalies, gravity interpolation, and mean gravity along the plumb line had to be modified for the measurements on the bridges and in the tunnel of the Link. Computed from form. (9.3) in Blakely (1996) gravity contributions from the tunnel or the pillars/pylons of the bridges are less than 1 mgal, therefore they have been ignored in all our computations. Consequently, points on the bridges have been considered as points in free air above the sea, i.e. case [c], cf. app.1, sect.4 - 6, on Peberholm as case [d], h_o=5 m, and in the tunnel as case [e], t_l>0, except for the endpoints, case [e], t_l<0. In addition, the point on top of the Telegraph Tower, as case [d], h_o=21.75 m.

Furthermore, ground density ρ=2.00 g/cm^3 has been used in all our gravity computations. As is seen from fig.6 ground conditions along the Oeresund Loop are more or less similar on both sides of the Sound, except for the Hälsingborg area, thus the traditional Danish density value above has been used in the entire region. Anyhow, the more common value ρ=2.67 could have been used, too, this does not really matter.

7.3 Bouguer anomalies applied. Fig. 6 is showing the Bouguer anomalies (ρ=2.67) available in the Oeresund region extracted from the NKG gravity database for gravity interpolation. Points from marine gravity as well as the new gravity points along the Link are not shown. As is seen the anomalies between Helsingør and Hälingborg are increasing about 24 mgal, which is indicating an uplift of the bedrock underneath Hälsingborg. In general, the anomalies on the Swedish side are growing in north-east direction, which is in good agreement with the north-west direction of the bedrock ridges Kullen (altitude 188 m) and Söderåsen (212 m).

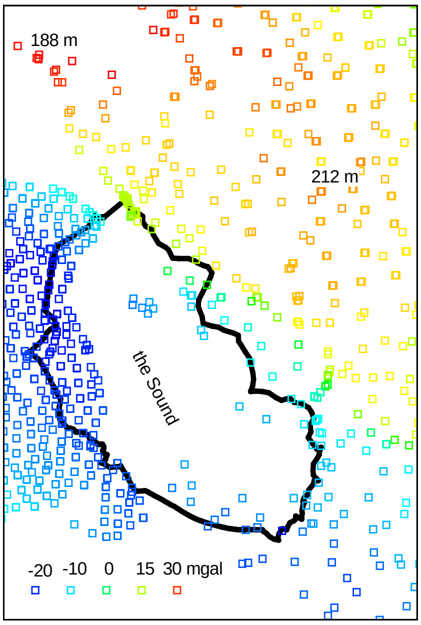


Fig. 6: Bouguer anomalies in the Oeresund region

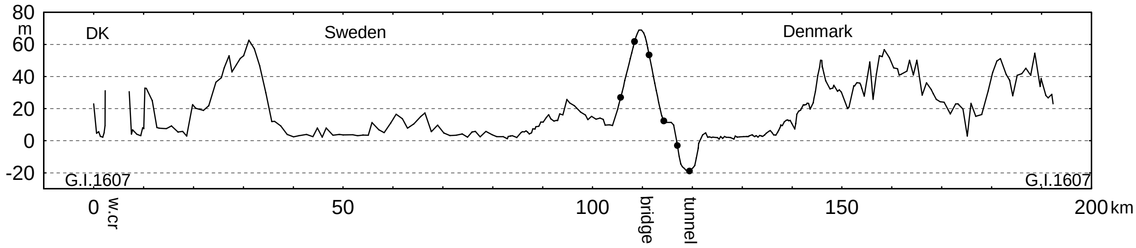
For the sake of consistency the anomalies in fig. 6 have been transformed into values corresponding to ρ=2.00, taking into account that the data points are either on the ground above the geoid or at the surface of the sea (from marine gravity), which is indicated by positive or negative heights in the database. The latter is referring to the (negative) depth of the sea. The transformation formula is given in app.1, form. (12). In addition, we have computed the Bouguer anomalies (ρ=2.00) of the new gravity points along the Link. Measured gravity and results are given in app.2, table A3.

7.4 Gravity interpolation. Applying the strict approach a gravity value is required at every leveling point. Based on the transformed Bouguer anomalies (ρ=2.00) mentioned in sect.7.3 gravity values have been found applying the GeoGrid program, cf. app.1, sect. 5.1, running the job 'nivtyng.bat'. Since ground density ρ=2.67 is used by the program the interpolated gravity values from GeoGrid had to modified (mod.1) according to ρ=2.00 using the formulas in app.1, sect.5.2. Furthermore, depending on the input height h any interpolation point is considered by the program either on the ground above the geoid (h>0) or at the surface of the sea (h<0, -h is the depth of the sea). That means applying the formulas, ibid., further modifications (mod.2) had to be done as regards the points of the Link and the point on top of the Telegraph Tower. Concerning the Link the final results g_P* are given in app.2, table A4.

7.5 Orthometric correction. Mean gravities $\overline{g}_P$ have been computed from the formulas in app.1, sect.6, using leveled heights. Hereafter, the orthometric corrections have been found from form. (6), app.1, using the average gravity value g_o=981500 mgal. The correction has been applied, too, to the optical water crossing, but not to the steel tape measurement.

In fig.8, bold curve, we have shown the differences between adjusted Helmert heights and the heights from an ordinary leveling adjustment starting at Hamlets Grav, where we have adopted the same initial height value. As is seen the impact of gravity on the adjusted heights is in the range of about ±1 mm. Also note the short segment (6km) north of Landskrona (Lk), where the differences are increased by almost 1½ mm. The jump at the beginning of the curve is coming from the orthometric correction (-0.76 mm) of the optical water crossing (w.cr. in fig.7). It is doubtful, if the computed value is strictly correct, since form. (6), app.1, is based on leveling with forward and backward sights of equal length, however, the sight lengths of the water crossing are extremely different, say, 50 m and 5 km. Disregarding the crossing and the segment mentioned above the curve behaves as expected, i.e. smoothly increasing from north to south and smoothly decreasing in the opposite direction, cf. app.1, sect.2.3. Nothing extraordinary is happening along the Link. Note also, the final value of the curve (-½mm) is indicating the deviation of the loop closing error applying gravity or not, cf. sect. 4.2.

figure 7: Height profile of the Oeresund Loop



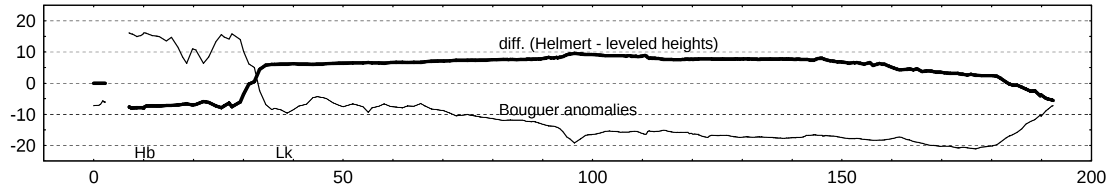


figure 8: Differences (Helmert heights – leveled heights) in 0.1mm vs. Bouguer anomalies in mgal

Additionally, we have shown in fig.8 the Bouguer anomalies (ρ=2.00) along the loop. Reflecting the depth of bedrock below the ground the anomalies are strongly increasing from Helsingør to the opposite side of the Sound, whereas a corresponding decrease is found along the line segment mentioned above.

8. Conclusions

- Applying the usual rejection limit for leveling to the measurement of the Link, it is fully comparable to the Danish Third Precise Leveling. Thus, it should be used for the connection of the new height networks in Denmark and Sweden.
- The suspicion that the optical water crossing (1980) is defective cannot be confirmed by the closing errors corrected for postglacial uplift, neither from the Oeresund Loop (-13mm), nor from the loop including the optical and hydrostatic crossing (-3mm).

Appendix 1: Leveling and Gravity

Normally, textbooks on leveling are only referring to leveling on the ground. In the following we try to substantiate the validity of the fundamental equations, cf. form. (3), (5), and (6) below, also for leveling on bridges or in tunnels. In this case, however, the common formulas for the computation of Bouguer anomalies, interpolated gravity, etc. have to be modified as shown.

1. Geopotential numbers and orthometric heights.

To move a particle of mass in a three-dimensional force field from one point to another the work required generally is depending on the path taken by the particle. However, as can be shown the earth's gravity acceleration **g** can be written as the gradient $(\partial W / \partial x, \partial W / \partial y, \partial W / \partial z)$ of a real function W in space, cf. Moritz and Heiskanen (1967), sect. 2-1

$$\mathbf{g} = \mathrm{grad}W \tag{1}$$

This is the fundamental equation for all what follows.

From (1) can be concluded due to the findings of Potential Theory that the work done by the gravity field is independent of the path. The function W is called the gravity potential of the earth, its value at the point P is the work done by the gravity field to move the unit mass from infinity to P, cf. Torge (1991), sect.2.1.1.

Due to (1) the work W_2-W_1 done by the gravity field to move the unit mass from point P_1 to P_2 is given by integrating the magnitude g of the gravity vector **g** along an arbitrary path connecting the points

$$W_2 - W_1 = \int_{P_2}^{P_1} g\,dn \tag{2}$$

Here dn is the elemental vertical displacement of differentially separated equipotential surfaces, W=constant, along the path, cf. Moritz and Heiskanen (1967), form. (4-4). The increment dn is counted positive in outwards direction, i.e. opposite to the gravity vector.

Let the geoid be given by the equipotential surface $W=W_o$ then the geopotential number C_P of any point P in space is defined by

$$C_P = W_o - W_P$$

Hence, C_P can be interpreted as the work done by the gravity field to move the unit mass from the point P to the geoid. The unit of geopotential numbers is 1g.p.u.=1kgal·meter=$10m^2/sec^2$ according to the unit of acceleration 1gal=$1cm/sec^2$.

A further consequence of (1) is that the gravity vector **g** at any point P is normal to the equipotential surface, $W=W_P$, through P. Hence, **g** is tangent to the plumb line through P intersecting normally (by definition) all equipotential surfaces. Let P_o denote the intersection point of the plumb line through P and the geoid $W=W_o$ then the orthometric height H_P of P is defined by the length of the plumb line section from P_o to P. Considering the latter as a path from P_o to P it follows from (2) that geopotential number and orthometric height are related as follows, cf. Moritz and Heiskanen (1967), form. (4-19), (4-20), (4-21)

$$C_P = \bar{g}_P H_P , \quad H_P = \frac{C_P}{\bar{g}_P} \tag{3}$$

where $\bar{g}_P$ (in kgal) is the mean value of gravity g integrated along the plumb line from P_o (H=0) to P ($H=H_P$)

$$\bar{g}_P = \frac{1}{H_P} \int_0^{H_P} g\,dH \tag{4}$$

As gravity approx. is 0.98 kgal in the area close to the earth's surface geopotential numbers in g.p.u. are about 2% smaller than the corresponding orthometric heights in meter.

2. Conversion of leveled height differences.

2.1 The inconsistency of leveled heights. According to above the elemental alteration dW of the potential W corresponding to a vertical elemental displacement dH at the point P is given by $dW = -g_P dH$, cf. Moritz and Heiskanen (1967), form. (2-14). It follows, since gravity g is varying on the equipotential surface through P, that the surface is nonparallel to the neighboring equipotential surfaces. According to chapter 4.1, ibid., the nonparallelism of the equipotential surfaces has the following consequences

- leveled height differences are depending on the leveling path between the endpoints
- summing up leveled height differences along a loop the resulting closing error is not strictly vanishing, even if the leveling has been performed without any kind of errors
- the usual observation equation for the adjustment of the leveled height difference from P_1 to P_2, $\Delta h_{12}=h_2-h_1$, is not strictly valid

That means, leveled heights do not constitute a consistent system. However, orthometric heights or geopotential numbers are free from the lacks above, thus, all leveling can be calculated strictly in the usual way, replacing leveled height differences by geopotential or orthometric height differences. From (3) orthometric heights can be computed from geopotential numbers, and vice versa. If metric heights are not needed, say in the computation of loop closing errors, it is often more practical to use geopotential differences.

2.2 Conversion into geopotential differences. As outlined in Moritz and Heiskanen (1967), chapter 4.1., leveled height differences are yielding differences of geopotential numbers (geopotential differences), if combined with gravity. Under normal conditions it holds with sufficient accuracy

$$\Delta C_{12} = C_2 - C_1 = \Delta h_{12} g_m \quad (5)$$

cf. Moritz and Heiskanen (1967), form. (4-3), see also the derivations in Torge (1991), sect. 4.3.5, and Jordan/Eggert/Kneissl (1956/1969), § 135. Above Δh_{12} is the leveled height difference and g_m (in kgal) is the arithmetic mean of gravity at the endpoints of the section. Studying the derivations in the textbooks mentioned before it can be seen that equation (5) is valid not only for leveling on the ground, but also for measurements on bridges or in tunnels.

2.3 The orthometric correction. Adding the orthometric correction OC_{12} to a leveled section height difference Δh_{12} the corresponding difference of orthometric heights $\Delta H_{12}=H_2-H_1$ is obtained. Form. (4-33) in Moritz and Heiskanen (1967) is giving the correction in case of leveling on the ground. As can be seen from its derivation it is valid, too, for leveling on bridges or in tunnels. In smaller regions at least, the formula can be written

$$OC_{12} = \frac{g_m - g_o}{g_o}\Delta h_{12} + \frac{\bar{g}_1 - g_o}{g_o} h_1 - \frac{\bar{g}_2 - g_o}{g_o} h_2 \quad (6)$$

Here g_m is again the arithmetic mean of gravity at the endpoints, g_o is an appropriate average value of gravity in the region, $\bar{g}$ is mean gravity defined through form. (4), and h is a preliminary height from leveling, e.g. In order to compute the correction mean gravity g_m has to be found from gravity interpolation, cf. sect.5, whereas $\bar{g}$ is calculated from sect.6. Strictly speaking, form. 6 is the so-called orthometric Helmert correction giving Helmert heights when added to leveled height differences.

In case of leveling on the ground above the geoid can be shown that the orthometric Helmert correction referring to ground density $\rho=2.00 g/cm^3$ can be simplified as follows

$$OC_{12} = -h_m(\Delta g_{12} + (0.3086 - 2\cdot 0.0838)\Delta h_{12})/g_o \quad \text{(in m)}$$

where h_m is the arithmetic mean of h_1 and h_2 (in m) and $\Delta g_{12}=g_2-g_1$ (in mgal) is the difference of gravity at the endpoints. As can be seen from the formula, in a low and flat area with smooth gravity the correction is summing up in the north-south direction, but hardly from east to west. Consequently, its impact on the closing error often is relatively small.

3. Gravity reduction by means of Bouguer plates.

Given the points P and Q on the same plumb line gravity reduction from P to Q is the prediction of gravity at Q from gravity at P. Gravity reduction is needed for the computation of mean gravity $\bar{g}$ as well as Bouguer anomalies Δg_B used for gravity interpolation. The simplest method is the simplified reduction of Poincaré and Prey, cf. Moritz and Heiskanen (1967), sect.4-3. Though the reduction seems rather rough it is generally working well for leveling in low and flat areas with smooth gravity.

Say, gravity has to be reduced from P to Q. For this purpose the masses along the plumb line through P and Q are replaced by Bouguer plates, i.e. infinite plane horizontal plates with constant mass density, such that both points are on the surface of a such a plate. Assuming P is below Q gravity at Q is obtained according to the following steps:

1) Remove all the plates above P and compute the corresponding gravity at P
2) Move P through free air to Q and compute the corresponding gravity at Q
3) Restore the plates removed and recomputed the gravity at Q.

An example is given below.

Obviously, removing a Bouguer plate located above/below a given point P is increasing/decreasing gravity at P by an amount corresponding to the plate's vertical attraction on P, whereas adding a plate above/below the point is decreasing/increasing gravity at P. According to Moritz and Heiskanen (1967), form. (3-15), the attraction is given by

$$A_B = c_\rho b \ \text{mgal}$$

where b (in meter) is the thickness of the plate and c_ρ is depending on the plate's mass density ρ (in g/cm^3). Thus, the attraction is independent from the distance of P from the plate. Common values of ρ and c_ρ are given below:

ground , standard value:	$\rho=2.67 g/cm^3$	$c_\rho=0.1119$
ground, Denmark:	$\rho=2.00$	$c_\rho=0.0838$
sea water:	$\rho=1.03$	$c_\rho=0.0432$

Note, due to the moraine and chalk densities of the Danish subsurface ground density ρ=2.00 is the traditional value for the computation of leveling in Denmark.

Finally, moving the point P to Q through free air the gravity change is given by

$$F = 0.3086\ell \text{ mgal}$$

where ℓ (in meter) is the distance between P and Q, counted negative if P is below Q.

□ Example: Assuming the point P is below the bottom of the sea (assumed plane) let h denote the height of P and d the depth of the sea. Obviously, since h is negative, t_l=-h-d is the positive height of the sea floor above P, cf. case [e] in sect.4. Furthermore, let P_l and P_o be the points, where the plumb line through P is intersecting the bottom and the surface of the sea. Now, let $s \in [0,t_l]$ be the distance along the plumb line through P, counted positive from P in upwards direction. Applying the procedure above (ρ=2.00) gravity g(s) at the point corresponding to s is predicted from gravity g_P at P by

$$g(s)=g_P+0.0432d+0.0838t_l -0.3086s+0.0838s -0.0838(t_l-s)-0.0432d = g_P+(2\cdot0.0838-0.3086)s \quad \text{(mgal)}$$

Correspondingly, counting the distance from P_l, $s_l=s-t_l \in [0,d]$ gravity $g(s_l)$ is predicted from gravity g_l at P_l

$$g(s_l) = g_l+0.0432d -0.3086s_l + 0.0432s_l - 0.0432(d-s_l) = g_l + (2\cdot0.0432 - 0.3086)s_l \quad \text{(mgal)}$$

Hence, applying $s=t_l$ and $s_l=d$ gravity at P_l and P_o is given by

$$g_l = g_P + (2\cdot0.0838 - 0.3086)t_l \quad \text{(mgal)}$$

$$g_o = g_l + (2\cdot0.0432 - 0.3086)d \quad \text{(mgal)}$$

□

4. Bouguer gravity and anomalies.

In order to make gravity g_P at a given point P comparable to what is 'normal' in accordance with a geometric/gravimetric model of the earth gravity is reduced from P to the intersection point P_o of the plumb line through P and the geoid, assuming a fictitious solid geoid with no masses above. The term 'solid' means, e.g., seawater is replaced by soil. The reduced value g_B is the Bouguer gravity of P and its deviation from 'normal' is the corresponding Bouguer anomaly Δg_B

$$\Delta g_B = g_B - \gamma_o \tag{7}$$

The 'normal' gravity value γ_o referring to the geodetic reference system GRS80 can be computed from form. (3.4) in Torge (1989), e.g. Obviously, Bouguer anomalies are reflecting the mass densities of the ground, which makes them suitable for gravity interpolation.

In the formulas below, derived in accordance with sect.3 applying ground density ρ=2.00, h is the height of the point P above the geoid/mean sea level and d is the depth of the sea at P's location. Bouguer gravity g_B is in mgal.

[a] P on the ground above geoid:

$$g_B= g_P - 0.0838h + 0.3086h \tag{8}$$

Replacing 0.0838 by 0.1119, cf. sect.3, (8) is corresponding to Moritz and Heiskanen (1967), form. (3-18). Obviously, the Bouguer plate between P and the geoid is removed, hereafter P is lowered to the geoid through free air. Similarly, we find:

[b] P at the surface of the sea:

$$g_B = g_P - 0.0432d + 0.0838d \tag{9}$$

[c] P in free air above the sea:

$$g_B = g_P - 0.0432d + 0.0838d + 0.3086h$$

[d] P in free air above the ground above geoid: (h_o is the height of P above ground)

$$g_B = g_P - 0.0838(h-h_o) + 0.3086h$$

[e] P below the surface of the sea: (t_l=-h-d is the height of the bottom above P)

P below the bottom, $t_l>0$: $g_B = g_P + 0.0838t_l + 0.0432d - 0.3086(-h) + 0.0838(-h)$

P above the bottom, $t_l<0$: $g_B = g_P - 0.0432(-t_l) + 0.0432(-h) - 0.3086(-h) + 0.0838d$

5. Gravity interpolation (GeoGrid)

5.1 The GeoGrid program. Gravity interpolation can be carried out by the GeoGrid program, which is part of the program package GRAVSOFT used for geoid computations, cf. Tscherning et al. (1992). Using ground density ρ=2.67 it proceeds as follows.

1. The interpolated Bouguer anomaly Δg_B* of the point P is computed by the weighted mean of Bouguer anomalies of 5 data points in each quadrant, which are closest to P. The weighting is done in accordance with the reciprocal squared distances of P from the data points applied (different options are available).
2. The corresponding Bouguer gravity g_B* is computed according to (7) using normal gravity of GRS80

$$g_B^* = \Delta g_B^* + \gamma_o$$

3. Interpolated gravity at P is now computed in two different ways depending on the input height of P.
   a. input height h>0: P is considered according to case [a] in sect.4, assuming the input height h is the height of the point considered. Accordingly, gravity g* at the point considered is calculated analogously to form. (8) reducing gravity g_B* from the fictitious geoid, cf. sect.4, to the point considered using ground density ρ=2.67g/cm³

$$g^* = g_B^* - 0.3086h + 0.1119h \tag{10}$$

b. input height h<0: P is considered according to case [b], assuming -h is the depth of the sea. Accordingly, gravity g* at the point considered is computed analogously to (9)

$$g^* = g_B^* - 0.1119(-h) + 0.0482(-h) \quad (11)$$

Obviously, to make sense the Bouguer anomalies of the data points should refer to ρ=2.67.

5.2 Modifications. For the sake of documentation the GeoGrid program has been used unchanged, however, a few minor modifications would have simplified significantly the following procedure of gravity interpolation.

As mentioned before ρ=2.00 is used for leveling in Denmark. Consequently, the Bouguer anomalies of the data points should refer to the same value. Given anomaly values Δg_B referring to ρ=2.67 are easily transformed into corresponding values $\mathit{\Delta} g_B$ referring to ρ=2.00. Regarding a data point P, case [a] or [b], it is immediately seen from formulas corresponding to (7), (8), and (9)

$$\text{[a]:} \qquad \mathit{\Delta} g_B = \Delta g_B + (0.1119-0.0838)h \quad (12)$$

The same formula goes for case [b] replacing h in the formula above by -d.

Now, what is happening, if GeoGrid is applying a data list of Bouguer anomalies (ρ=2.00)? Let $\mathit{\Delta} g_B^*$ be the resulting interpolated Bouguer anomaly of the point P and g^* the corresponding outcome from GeoGrid. Since the data list is referring to ρ=2.00 GeoGrid should have used the same value in the computation of g^*. Taking this into account (modification 1) it is seen from (10) and (11) that no matter of the sign of the input height h the consistent gravity value at the point considered by GeoGrid is

$$g^{**} = g^* + (0.0838-0.1119)h \quad (13)$$

Below, we are giving gravity g_P^* (in mgal) consistently interpolated based on ρ=2.00 at the point P depending on the different cases in sect.4. It is assumed the GeoGrid input height of P is the height h of P, except in case [b], where the input height -d is assumed. As is evident from above

$$\text{[a], [b]:} \qquad g_P^* = g^{**}$$

In the other cases further steps are needed, since gravity g^{**} has to be reduced from the point considered by GeoGrid to the point P (modification 2). Below we are continuing the example in sect.3, i.e. case [e], $t_l>0$.

□ <u>Example (cont.):</u> Since the height of point P is negative, g^{**} is gravity at the point considered, i.e. the point P_o, assuming -h is the depth of the sea, cf. 3.b. in sect.5.1. Hence, interpolated gravity g_P^* at P is obtained removing the fictitious sea water plate from P to P_o, lowering P_o to P through free air, and adding the plates of soil and water from P to P_l and P_l to P_o, respectively. Hence

$$\text{[e], } t_l>0: \qquad g_P^* = g^{**} - 0.0432(-h) + 0.3086(-h) - 0.0838t_l - 0.0432d \qquad □$$

Similarly, we find

$$\text{[e], } t_l<0: \qquad g_P^* = g^{**} - 0.0432(-h) + 0.3086(-h) - 0.0838(-t_l) + 0.0432(-t_l) - 0.0432(-h)$$

$$\text{[c]:} \qquad g_P^* = g^{**} - 0.0838(h+d) + 0.0432d$$

$$\text{[d]:} \qquad g_P^* = g^{**} - 0.0838h_o$$

6. The computation of mean gravity $\overline{g}$.

Mean gravity is needed for the transformation of geopotential numbers into orthometric heights, or vice versa, cf. (3). It is also needed for the computation of orthometric corrections, cf. (6). According to (4) mean gravity $\overline{g}_P$ corresponding to the point P is determined from integration of gravity g along the plumb line from the geoid to P. However, this gravity is normally unknown. The Helmert approach to solve the problem is to replace the masses along the plumb line by corresponding Bouguer plates. By this, gravity between the geoid and the point P becomes a piecewise linear function of distance in accordance with the different plates. Thus, mean gravity along the section of the plumb line cut out by a given plate, is easily found from the arithmetic mean of gravity at the section's endpoints.

Corresponding to the different cases in sect.4 we are giving below mean gravity $\overline{g}_P$ (in mgal) based on ρ=2.00 of the point P, H_P is the orthometric height of P. To make the procedure more comprehensible we are continuing the example in sect.3.

□ <u>Example (cont.):</u> Splitting up the integration according to the plates of soil and sea water between P and P_o mean gravity $\overline{g}_P$ can be written

$$\overline{g}_P = -(I_1 + I_2)/H_P$$

Here $I_1 = \int_0^{t_l} g(s)ds$ and $I_2 = \int_{t_l}^{-H_P} g(s)ds$. As is seen from the example g(s) is a piecewise linear function of s in accordance with the plates. Thus, $I_1=½(g_P+g_l)t_l$, $I_2=½(g_l+g_o)d$, hence

$$\text{[e], } t_l>0: \qquad \overline{g}_P = -\frac{½(g_P + g_l)t_l + ½(g_l + g_o)d}{H_P}$$

Gravity g_P is often found by interpolation, whereas g_o and g_1 are given in sect.3. □

Similarly, mean gravity corresponding to P in all other cases is given by

[e], $t_1<0$: $\bar{g}_P = g_P + ½(0.3086 - 2·0.0432)H_p$

[a]: $\bar{g}_P = g_P + ½(0.3086 - 2·0.0838)H_p$

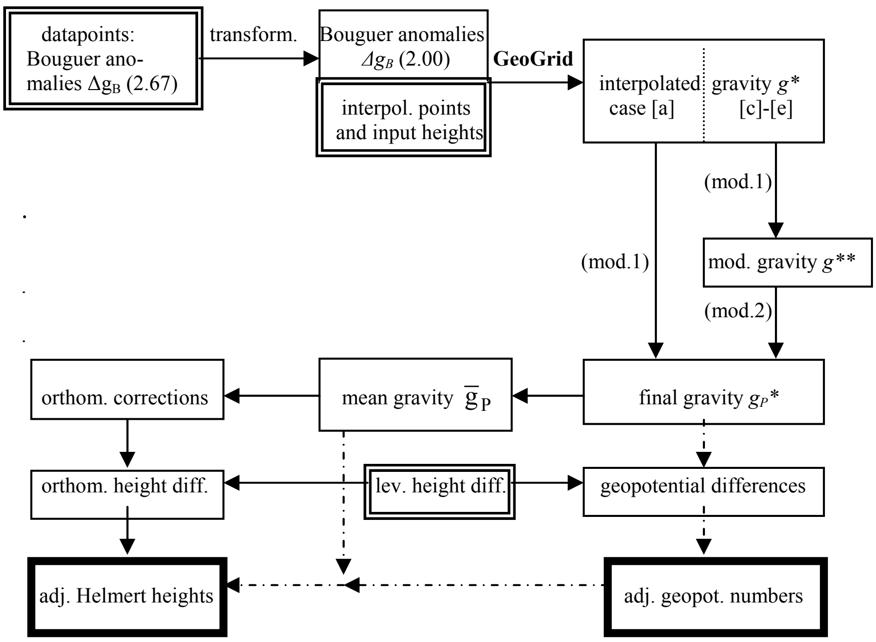
[c]: $\bar{g}_P = g_P + ½·0.3086H_p$

[d]: $$\bar{g}_P = \frac{½(g_o + g_1)(H_P - h_o) + ½(g_1 + g_P)h_o}{H_P}$$

where g_o is gravity at the geoid below P and g_1 is corresponding to the ground , i.e.

$$g_o = g_1 + (0.3086 - 2·0.0838)(H_P - h_o)$$
$$g_1 = g_P + 0.3086h_o$$

7. Dataflow. The diagram below is illustrating the entire dataflow during gravity interpolation and leveling adjustment. The dot-and-dash lines are irrelevant, since geopotential differences have not been used. The denotation is in accordance with previous sections.



Appendix 2

1. Stability investigation in the Helsingør region

According to the set up in table 1 we have to look for a stable subsoil benchmark leveled in 1980 and 1992. Since most of the subsoil points have been omitted in the leveling from 1992 the number of candidates can be narrowed down to the following: Kvistgård (G.I.1646, G.I.1647, G.I.1648), Hamlets Grav (G.I.1605, G.I.1606, G.I.1607), and G.M.1341. The point last-mentioned is unsuitable for connection, it is located on a bastion of the Kronborg Castle and a subsidence of about 35mm relative to Hamlets Grav has taken place from 1940 to 1992. Due to numerous surveying campaigns in the area precise leveling has been conducted frequently: 1898 (First Precise Leveling), 1908, 1939, 1940, 1941, 1942, 1943 (Second Precise Leveling), 1980, 1983, and 1992 (Third Precise Leveling). However, in order to evaluate stability between and within the groups Hamlets Grav and Kvistgård, cf. fig. 4, only the levelings from 1941 on are relevant, simply because the groups did not exist before 1940 and 1941, respectively. The data sources applied are indicated in table A1, where the pages are referring to computation volumes of G.I. For the sake of completeness we also give the references of the oldest levelings: 1898(#DK_niDprs), 1908(p.142 ff.), 1939(p.2058 ff.).

1.1 Leveling within the groups of Hamlets Grav and Kvistgård. In table A1 we have listed all measurements available. The results recorded are means of 4 to 9 single runs with sight lengths of about 35m.

Hamlets Grav:		1940	1941	1942	1943	1980	1983	1992
	G.I.1605							
		-0.55719	-0.55692	-0.55673	-0.55652	-0.55799	-0.55809	-0.55776
	G.I.1606							
		-0.89963	-0.89914	-0.89946	-0.89966	-0.89785	-0.89804	-0.89827
	G.I.1607	-----------	-----------	-----------	-----------	-----------	-----------	-----------
		-1.45682	-1.45606	-1.45619	-1.45618	-1.45584	-1.45613	-1.45603
	source:	p.2077 ff.	p.2186 ff	p.2366 ff	p.7709 ff	#DK_niDniv	#DK_niDniv	#DK_niDprs

Kvistgård:		1941	1942	1943	1980	1983	1992
	G.I.1646						
		-0.96802	-0.96806	-0.96804	-0.96890	-0.96902	-0.96894
	G.I.1647						
		-0.75873	-0.75860	-0.75872	-0.75801	-0.75786	-0.75654
	G.I.1648	-----------	-----------	-----------	-----------	-----------	-----------
		-1.72675	-1.72666	-1.72676	-1.72691	-1.72688	-1.72548
	source:	#DK_niDprs	#DK_niDprs	#DK_niDprs	#DK_niDniv	#DK_niDniv	#DK_niDprs

table A1: Repeated levelings within the groups

Considering the group Hamlets Grav we can conclude that G.I.1605 and G.I.1607 can be considered mutually stable from 1941 to 1992, whereas G.I.1606 has subsided about 1½mm during the years from 1943 to 1980, hereafter the point seems stable. That means any one of the three benchmarks could be used for the connection 1980/1992. As regards the Kvistgård group we notice mutual stability of G.I.1646 and G.I.1647 from 1980 on, whereas G.I.1648 has raised about 1½mm (according to the field books this is probably caused by some construction work close to the point).

1.2 Levelling between the groups. Below, we are giving the levelings available from Kvistgård (G.I.1647) to Hamlets Grav (G.I.1607), which are including the old subsoil benchmark G.M.1337 from the First Precise Leveling. The results recorded are rounded measured height differences summed up along the shortest route connecting the groups. The same data sources as above have been used.

	1940	1941	1942	1943	1980	1983	1992
G.I.1647							
4.5km		-4.513	-4.516	-4.515	-4.517	-4.516	-
G.M.1337							
4.5km	-16.278	-16.276	-16.279	-16.277	-16.267	-16.272	-
G.I.1607		----------	----------	-----------	----------	----------	
		-20.790	-20.794	-20.792	-20.784	-20.788	-20.789

table A2: Repeated levelings between the groups

From the table we can conclude that G.I.1647 and G.M.1337 seem mutually stable from the forties up to the eighties, whereas in 1980 there is an apparent raise of G.I.1607 of about 10mm relative to G.M.1337. Since this could not be explained neither by a gross leveling error nor by postglacial uplift a new leveling was carried out in 1983, now indicating a subsidence of about 5mm since 1980. The validity of these contradicting results seems rather unlikely. However, since the result from 1983 is confirmed in 1992 we have decided to ignore the leveling from 1980 as much as possible. This is excluding the Kvistgård benchmarks for the connection 1980/1992, thus, the benchmark G.I.1607, which is directly entering the leveling line, has been selected as connection point. As is seen from the table there might be a raise of a few millimeters of G.I.1607 relative to G.I.1647 during the period from the forties up to the nineties. This is in good agreement with the postglacial land uplift according to Ekman (1996).

The reason for the suspicious deviation of the height difference in 1980 from G.M.1337 and G.I.1607 can hardly be found today, but perhaps it should be mentioned that most of the leveling sections in 1980 were measured by three single runs according to the field book GA.I, journal bind 102. That means systematic errors might not be fully removed from the means, which have been used in the computation of the height difference in question.

2. Gravity along the Link.

2.1 Gravity and Bouger anomalies of the new gravity points, ρ=2.00. The gravity profile of the Link has been measured relative to the KMS gravity station 120 in the beginning of April 2000 by two LaCoste Romberg gravimeters, G867 and G466. The location of the new points can be seen from the bold points in fig.7. Measured gravity g_P and corresponding Bouguer anomalies Δg_B computed from the formulas in app.1, sect.4, are given below. Normal gravity γ_o has been computed as mentioned in app.1, sect.4. The leveled heights h of the points in the tunnel are including an offset of 65 cm, since gravity was measured at the bottom of tunnel. The depth d of the sea has been estimated roughly from a marine map.

point no.	loc.	g_P	h	d	Δg_B
4078/432 661	EastBridge	981 531.40 mgal	26.84 m	2 m	-15.76 mgal
4078/430 661	East Bridge	521.17	61.71	7	-15.50
4078/411 061	West Bridge	524.65	53.46	7	-15.36
4078/301 119	Peberholm	539.63	12.34	-	-15.14
4078/300 501	Tunnel	543.93	-3.70	3	-15.90
4078/919	Tunnel	547.19	-19.59	9	-16.24

table A3: Measured gravity and Bouguer anomalies (ρ=2.00) of the gravity profile of the Link

2.2 Interpolated gravity g_P*, ρ=2.00. The gravity values in table A4 have been computed according to app.1, sect 5.2. Bold point numbers are referring to stable points kept in the KMS point database. The last column is indicating points, which have been measured by one national team only.

	point P	h	d	g_P*	
Bridge	**4078/500 503**	19.48 m	2 m	981533.88 mgal	
	4078/500 502	21.99	2	981532.95	Sw
	4078/500 501	22.00	2	981532.97	
	4078/500 501.1	22.00	2	981532.97	DK
	4078/432 861	23.86	2	981532.33	Sw
	4078/432 661	26.84	2	981531.40	
	4078/432 461	30.34	2	981530.36	
	4078/432 261	33.81	2	981529.34	
	4078/432 061	37.35	3	981528.24	DK
	4078/431 961	37.40	3	981528.23	Sw
	4078/431 861	40.82	3	981527.22	
	4078/431 661	44.26	4	981526.18	
	4078/431 461	47.76	5	981525.12	
	4078/431 261	51.22	5	981524.18	
	4078/431 061	54.75	6	981523.16	
	4078/430 861	58.22	6	981522.20	
	4078/430 661	61.71	7	981521.17	
	4078/430 461	64.78	7	981520.28	
	4078/430 261	67.07	7	981519.57	
(pylon)	**4078/400 506**	68.88	7	981518.94	
(pylon)	**4078/400 502**	68.91	7	981518.71	
	4078/410 261	67.13	7	981519.15	
	4078/410 461	64.78	7	981519.77	
	4078/410 661	61.70	7	981521.01	
	4078/410 861	57.82	7	981522.95	
	4078/411 061	53.46	7	981524.65	
	4078/411 261	49.03	7	981526.02	
	4078/411 461	44.66	6	981527.42	
	4078/411 661	40.25	6	981528.83	
	4078/411 861	35.88	5	981530.34	
	4078/412 061	31.48	5	981531.82	
	4078/412 261	27.74	5	981533.13	
	4078/412 461	23.94	5	981534.44	
	4078/413 261	20.28	4	981535.78	Sw
Bridge	**4078/300 511**	15.63	4	981537.51	
Peberholm	4078/301 119	12.34	-	981539.63	
	4078/301 047	11.40	-	981539.95	
	4078/301 011	11.38	-	981540.01	
Peberholm	4078/300 975	9.78	-	981540.48	
Tunnel	**4078/300 502**	-0.64	3	981543.29	
	4078/300 501	-3.05	3	981543.85	(to be continued)

(continued)

	4078/1 409	-9.56	3	981544.93
	4078/1 919	-14.61	5	981545.91
	4078/1 719	-16.30	5	981546.31
	4078/1 519	-17.31	9	981546.77
	4078/1 319	-18.38	9	981547.08
	4078/1 119	-19.44	9	981547.47
	4078/919	-18.94	9	981547.09
	4078/719	-17.90	9	981547.16
	4078/519	-16.86	9	981546.96
	4078/319	-15.50	5	981546.77
	4078/119	-10.78	3	981546.13
	4078/100 502	-4.61	2	981545.30
Tunnel	**4078/100 501**	-1.80	2	981544.89

table A4: Interpolated gravity (ρ=2.00) along the Link

●●●

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Double runs of the Oeresund Loop

Hamlets Grav - Pålsjø Cliff			
G.I.1607	K -06-09006	555	-18.40051
K -06-09006	K -06-09034	430	0.99555
K -06-09034	K -06-00087	344	-2.91636
K -06-00087	K -06-00053	518	-0.46586
K -06-00087	K -06-00053	534	-0.46664
K -06-00053	K -06-09168	217	2.55088
K -06-09168	K -06-09008	227	4.57693
K -06-09008	K -06-09170	22	21.7468
K -06-09170	032*2*3116	4800	-0.5903
Pålsjø Cliff - East Bridge			
032*2*3116	032*2*3102	476	-26.42334
032*2*3102	032*1283*0713	197	2.57716
032*1283*0713	032*1283*0953	880	-2.74134
032*1283*0953	032*1283*0922	778	-0.80837
032*1283*0922	032*1283*0703	361	4.80158
032*1283*0703	032*1283*1032	250	-0.47496
032*1283*1032	032*1*3124	196	25.30183
032*1*3124	032*1*3119	295	-0.23249
032*1*3119	032*1*3120	1189	-7.57831
032*1*3120	032*1*3121	965	-16.81553
032*1*3121	032*9999*3121	429	-0.36191
032*9999*3121	032*1*3123	1418	-0.26218
032*1*3123	032*2*2101	1050	1.60767
032*2*2101	032*1*2104	1287	-3.72564
032*1*2104	032*1*2105	941	0.31725
032*1*2105	032*1*2106	830	-2.86912
032*1*2106	032*1*2107	1167	19.63731
032*1*2107	032*1*2206	624	-2.24019
032*1*2206	032*1*1204	1569	-1.47465
032*1*1204	032*1*1210	1047	2.94908
032*1*1210	032*1*1211	1432	14.73561
032*1*1211	032*1*1212	1103	2.72038
032*1*1212	032*1*1213	523	6.02802
032*1*1213	032*1*1309	1009	7.69969
032*1*1309	032*11*1317	576	-9.98954
032*11*1317	032*1*1311	1687	8.36616
032*1*1311	032*1*1316	660	1.66891
032*1*1316	032*11*0309	1081	9.63501
032*11*0309	032*1*0205	1084	-5.36118
032*1*0205	032*1*0206	1048	-10.48983
032*1*0206	032*11*0210	1268	-16.94041
032*11*0210	032*1*0209	1155	-17.83328
032*1*0209	032*1*0207	592	0.01595
032*1*0207	022*1*9204	1236	-2.78429
022*1*9204	022*1*9205	1236	-5.30230
022*1*9205	022*1*9309	1334	-1.55838
022*1*9309	022*1*9310	1031	0.71776
022*1*9310	022*9999*9304	1531	0.83602
022*9999*9304	022*1*9312	1294	-1.41624
022*1*9312	022*1*9313	898	5.43764
022*1*9313	022*1*8303	997	-5.72160
022*1*8303	022*1*8304	813	5.73844
022*1*8304	022*1*8305	1318	-4.60347
022*1*8305	022*1*8306	1278	0.59006
022*1*8306	022*1*8409	790	-0.31506
022*1*8409	022*1*7409	1887	0.09927
022*1*7409	022*1*7408	982	-0.57922
022*1*7408	022*1*7303	1111	0.40551
022*1*7303	022*1*7301	1048	-0.17979
022*1*7301	022*9999*7410	714	7.87950
022*9999*7410	022*1*7403	1368	-4.45244
022*1*7403	022*1*7404	1079	-1.82808
022*1*7404	022*1*6406	1344	6.07948
022*1*6406	022*1*6405	1095	5.42284
022*1*6405	022*11*6407	1253	-2.81875
022*11*6407	022*1*6517	989	-5.87114
022*11*6407	022*1*6517	987	-5.87151
022*1*6517	022*1*6518	1298	2.72412
022*1*6517	022*1*6518	1290	2.72445
022*1*6518	022*1*6519	1401	4.56417
022*1*6519	022*1*6515	799	2.28259
022*1*6515	022*1*6516	1292	-11.75967
022*1*6516	022*1*5507	1266	4.07845
022*1*5507	022*1*5508	1261	-4.85728
022*1*5508	022*1*5509	1204	-1.55751
022*1*5509	022*1*5510	1247	0.17462
022*1*5510	022*1*5511	1228	0.83019
022*1*5511	022*2*5505	1052	-2.05822
022*2*5505	022*1*5512	886	3.24222
022*1*5512	022*2*4501	685	0.32640
022*2*4501	022*1*4505	958	-3.45540
022*1*4505	022*1*4506	1170	3.48181
022*1*4506	022*1*4507	1248	-2.04387
022*1*4507	022*1*4508	924	-1.22395
022*1*4508	022*2*4504	1272	-0.02258
022*2*4504	022*1*4509	816	-1.29970
022*1*4509	022*1280*0764	162	1.40626
022*1280*0764	022*1280*0959	455	0.21849
022*1280*0959	022*1*3506	419	0.31138
022*1*3506	022*1280*0958	872	-1.12392
022*1280*0958	022*1*3507	859	3.02548
022*1*3507	022*1280*0080	300	0.64929
022*1280*0080	022*1*3508	336	-0.12420
022*1*3508	022*1280*0288	379	0.63550
022*1280*0288	022*1*3509	265	-0.81723
022*1*3509	022*1280*0141	401	-1.10018
022*1280*0141	022*1*3510	431	0.66180
022*1*3510	022*1280*0142	244	2.50207
022*1280*0142	022*1*3511	460	-0.12100
022*1*3511	022*1280*0255	164	1.26987
022*1280*0255	022*1*3401	591	0.45717
022*1*3401	022*1280*0249	385	2.45994
022*1280*0249	022*1*3402	487	0.01928
022*1*3402	022*1280*0465	643	2.87132
022*1280*0465	022*1*2425	464	1.85279
022*1*2425	022*1280*0956	435	-2.65869
022*1280*0956	022*2*2414	642	-1.37452
022*2*2414	022*1280*0477	525	0.40207
022*1280*0477	022*1280*0914	193	-0.01146
022*1280*0914	022*1*2422	388	4.15367
022*1*2422	022*1*2421	675	-0.64702
022*1*2421	022*1*2420	813	9.48434
022*1*2420	022*1*2419	556	-2.03922
022*1*2419	022*1*2418	913	-1.84510
022*1*2418	022*1*2417	1286	-3.84499
022*1*2417	022*1*2423	939	-1.86249
022*1*2423	022*9999*2423	492	-2.89425
022*9999*2423	022*46*200004	756	2.04942
022*46*200004	022*1*2311	857	-1.78011
022*1*2311	022*1*2310	818	0.62461
022*1*2310	022*46*200003	648	-0.70353
022*46*200003	022*1*2309	367	-3.60825
022*1*2309	022*1*2308	916	0.21854
022*1*2308	022*46*200001	609	-0.38980
022*46*200001	4078/500 503	978	9.96653
East Bridge - High Bridge			
4078/500 503	4078/500 501	225	2.52102
4078/500 503	4078/500 501.1	274	2.51802
4078/500 502	4078/500 501	81	0.00201
4078/500 501	4078/432 661	389	4.83744
4078/500 501.1	4078/432 661	394	4.84160
4078/432 661	4078/432 461	281	3.50224
4078/432 661	4078/432 461	279	3.50303
4078/432 461	4078/432 261	280	3.46916
4078/432 461	4078/432 261	278	3.46932
4078/432 261	4078/432 061	277	3.53847
4078/432 261	4078/432 061	275	3.53928
4078/432 061	4078/431 861	282	3.46901
4078/432 061	4078/431 861	281	3.46984
4078/431 861	4078/431 661	278	3.44458
4078/431 661	4078/431 461	279	3.50287
4078/431 661	4078/431 461	280	3.50370
4078/431 461	4078/431 261	281	3.45594
4078/431 461	4078/431 261	281	3.45666
4078/431 261	4078/431 061	283	3.52902
4078/431 261	4078/431 061	280	3.52872
4078/431 061	4078/430 861	280	3.46970
4078/431 061	4078/430 861	280	3.47021
4078/430 861	4078/430 661	279	3.48958
4078/430 861	4078/430 661	281	3.48961
4078/430 661	4078/430 461	279	3.06753
4078/430 661	4078/430 461	279	3.06866
4078/430 461	4078/430 261	276	2.29430
4078/500 503	4078/500 502	227	2.51847

4078/500 503	4078/500 501	224	2.52099
4078/500 503	4078/500 501.1	224	2.51856
4078/500 501	4078/432 861	159	1.86373
4078/500 501.1	4078/432 861	148	1.86723
4078/432 861	4078/432 661	242	2.97255
4078/432 861	4078/432 661	253	2.97289
4078/432 661	4078/432 461	282	3.50221
4078/432 661	4078/432 461	282	3.50208
4078/432 461	4078/432 261	282	3.46879
4078/432 261	4078/431 961	285	3.59686
4078/432 261	4078/431 961	285	3.59670
4078/431 961	4078/431 861	280	3.41143
4078/431 961	4078/431 861	280	3.41133
4078/431 861	4078/431 661	282	3.44495
4078/431 861	4078/431 661	282	3.44450
4078/431 661	4078/431 461	282	3.50264
4078/431 661	4078/431 461	282	3.50300
4078/431 661	4078/431 261	563	6.95879
4078/431 461	4078/431 261	282	3.45670
4078/431 461	4078/431 261	282	3.45619
4078/431 261	4078/431 061	284	3.52845
4078/431 261	4078/431 061	283	3.52812
4078/431 261	4078/430 861	564	6.99879
4078/431 061	4078/430 861	282	3.47004
4078/431 061	4078/430 861	282	3.47036
4078/430 861	4078/430 661	282	3.48938
4078/430 861	4078/430 661	282	3.48955
4078/430 861	4078/430 461	563	6.55795
4078/430 661	4078/430 461	282	3.06779
4078/430 461	4078/430 261	353	2.29429
4078/430 461	4078/430 261	280	2.29449
High Bridge - West Bridge			
4078/430 261	4078/400 506	306	1.80896
4078/430 261	4078/400 506	303	1.80869
4078/400 506	4078/400 502	529	0.02962
4078/400 506	4078/400 502	490	0.02833
4078/400 502	4078/410 261	434	-1.77911
4078/400 502	4078/410 261	301	-1.77923
4078/430 261	4078/400 506	307	1.80986
4078/430 261	4078/400 506	306	1.80885
4078/430 261	4078/400 506	307	1.81112
4078/400 506	4078/400 502	495	0.02905
4078/400 502	4078/410 261	304	-1.77968
4078/400 502	4078/410 261	304	-1.78008
4078/400 502	4078/410 261	305	-1.77994
4078/400 502	4078/410 261	304	-1.77933
4078/400 502	4078/410 261	304	-1.77955
West Bridge - Peberholm			
4078/410 261	4078/410 461	278	-2.35130
4078/410 261	4078/410 461	278	-2.35010
4078/410 461	4078/410 661	277	-3.07664
4078/410 661	4078/410 861	279	-3.88796
4078/410 661	4078/410 861	281	-3.88824
4078/410 861	4078/411 061	280	-4.35990
4078/410 861	4078/411 061	281	-4.35970
4078/411 061	4078/411 261	282	-4.42618
4078/411 061	4078/411 261	283	-4.42741
4078/411 261	4078/411 461	281	-4.36659
4078/411 261	4078/411 461	279	-4.36644
4078/411 461	4078/411 661	283	-4.40889
4078/411 461	4078/411 661	280	-4.40904
4078/411 661	4078/411 861	283	-4.37433
4078/411 661	4078/411 861	280	-4.37394
4078/411 861	4078/412 061	282	-4.39739
4078/411 861	4078/412 061	279	-4.39711
4078/412 061	4078/412 261	244	-3.73762
4078/412 061	4078/412 261	239	-3.73690
4078/412 261	4078/412 461	245	-3.80004
4078/412 261	4078/412 461	245	-3.80019
4078/412 461	4078/300 511	581	-8.31633
4078/410 261	4078/410 461	280	-2.35134
4078/410 261	4078/410 461	280	-2.34970
4078/410 261	4078/410 661	564	-5.42706
4078/410 261	4078/410 661	561	-5.42764
4078/410 461	4078/410 661	282	-3.07612
4078/410 461	4078/410 661	282	-3.07555
4078/410 661	4078/410 861	282	-3.88833
4078/410 661	4078/410 861	281	-3.88822
4078/410 661	4078/411 061	563	-8.24807
4078/410 861	4078/411 061	282	-4.35942
4078/410 861	4078/411 061	282	-4.35910
4078/411 061	4078/411 261	283	-4.42647
4078/411 061	4078/411 261	283	-4.42615
4078/411 061	4078/411 461	564	-8.79287
4078/411 261	4078/411 461	282	-4.36598
4078/411 261	4078/411 461	282	-4.36538
4078/411 461	4078/411 661	282	-4.40922
4078/411 461	4078/411 661	282	-4.40865
4078/411 461	4078/411 861	563	-8.78234
4078/411 461	4078/411 861	563	-8.78378
4078/411 661	4078/411 861	280	-4.37374
4078/411 661	4078/411 861	281	-4.37335
4078/411 861	4078/412 061	282	-4.39805
4078/411 861	4078/412 061	282	-4.39694
4078/411 861	4078/412 261	524	-8.13461
4078/411 861	4078/412 261	523	-8.13487
4078/412 061	4078/412 261	242	-3.73727
4078/412 061	4078/412 261	242	-3.73695
4078/412 261	4078/412 461	245	-3.80009
4078/412 261	4078/412 461	245	-3.79952
4078/412 261	4078/413 261	479	-7.46441
4078/412 261	4078/413 261	479	-7.46470
4078/412 461	4078/413 261	234	-3.66453
4078/412 461	4078/413 261	250	-3.66388
4078/413 261	4078/300 511	347	-4.65054
4078/413 261	4078/300 511	347	-4.65112
4078/413 261	4078/300 511	349	-4.65219
Peberholm - Tunnel			
4078/300 511	4078/301 119	482	-3.28364
4078/301 119	4078/301 083	478	-0.96298
4078/301 119	4078/301 083	479	-0.96315
4078/301 083	4078/301 047	481	0.01445
4078/301 083	4078/301 047	481	0.01444
4078/301 047	4078/301 011	478	-0.01556
4078/301 047	4078/301 011	482	-0.01567
4078/301 011	4078/300 975	481	-1.60287
4078/301 011	4078/300 975	479	-1.60311
4078/300 975	4078/300 502	669	-10.41639
4078/300 975	4078/300 502	668	-10.41640
4078/300 502	4078/300 501	92	-2.41440
4078/300 502	4078/300 501	96	-2.41390
4078/300 511	4078/301 119	482	-3.28277
4078/300 511	4078/301 119	484	-3.28357
4078/301 119	4078/301 083	481	-0.96399
4078/301 119	4078/301 083	482	-0.96373
4078/301 083	4078/301 047	485	0.01408
4078/301 083	4078/301 047	482	0.01438
4078/301 083	4078/301 047	481	0.01445
4078/301 047	4078/301 011	493	-0.01670
4078/301 047	4078/301 011	481	-0.01630
4078/301 047	4078/301 011	482	-0.01674
4078/301 011	4078/300 975	484	-1.60312
4078/301 011	4078/300 975	482	-1.60309
4078/300 975	4078/300 502	676	-10.41739
4078/300 975	4078/300 502	668	-10.41721
4078/300 502	4078/300 501	99	-2.41405
4078/300 502	4078/300 501	94	-2.41337
4078/300 502	4078/300 501	95	-2.41375
4078/300 502	4078/300 501	96	-2.41406
4078/300 502	4078/300 501	94	-2.41392
Tunnel - Copenhagen			
4078/300 501	4078/1 409	331	-6.50129
4078/300 501	4078/1 409	272	-6.50114
4078/1 409	4078/1 919	356	-5.05040
4078/1 409	4078/1 919	357	-5.05012
4078/1 919	4078/1 719	350	-1.68694
4078/1 919	4078/1 719	351	-1.68650
4078/1 719	4078/1 519	352	-1.01608
4078/1 719	4078/1 519	352	-1.01654
4078/1 519	4078/1 319	348	-1.07300
4078/1 519	4078/1 319	349	-1.07318
4078/1 319	4078/1 119	352	-1.06228
4078/1 319	4078/1 119	351	-1.06185
4078/1 119	4078/919	350	0.50049
4078/1 119	4078/919	350	0.50024
4078/919	4078/719	348	1.04203
4078/919	4078/719	350	1.04168
4078/719	4078/519	347	1.04412
4078/719	4078/519	349	1.04400
4078/519	4078/319	348	1.35837

4078/519	4078/319	351	1.35806	K -01-06429	K -01-08057	290	-0.64472
4078/319	4078/119	349	4.71790	K -01-08057	K -01-08175	225	0.49246
4078/319	4078/119	352	4.71824	K -01-08175	K -01-06400	150	0.10273
4078/119	4078/100 502	351	6.17122	K -01-06400	K -01-06382	336	-0.96548
4078/119	4078/100 502	343	6.17069	K -01-06382	K -01-06346	298	1.12842
4078/100 502	4078/100 501	107	2.80724	K -01-06346	K -01-06335	207	-0.18827
4078/100 502	4078/100 501	108	2.80674	K -01-06335	K -01-09038	172	-0.20341
4078/300 501	4078/1 409	274	-6.50110	K -01-09038	K -01-09037	254	-0.22816
4078/300 501	4078/1 409	268	-6.50124	K -01-09037	K -01-06272	411	0.99371
4078/1 409	4078/1 919	358	-5.05049	K -01-06272	K -01-06261	288	0.89364
4078/1 409	4078/1 919	358	-5.05047	K -01-06261	K -01-07657	559	1.36449
4078/1 409	4078/1 919	359	-5.05054	K -01-07657	K -01-06682	277	0.34901
4078/1 919	4078/1 719	353	-1.68648	K -01-06682	K -01-06179	304	-1.30860
4078/1 919	4078/1 719	353	-1.68687	K -01-06179	K -01-08018	331	-1.33861
4078/1 919	4078/1 719	352	-1.68687	K -01-08018	K -01-06799	445	-0.08885
4078/1 719	4078/1 519	354	-1.01576	K -01-06799	K -01-06109	379	1.75979
4078/1 719	4078/1 519	353	-1.01575	K -01-06109	G.M.1385/1386	186	1.13816
4078/1 719	4078/1 519	354	-1.01574	G.M.1385/1386	K -01-06110	255	1.52328
4078/1 519	4078/1 319	351	-1.07299	K -01-06110	G.M.1384/1385.1	150	1.19568
4078/1 519	4078/1 319	351	-1.07298	G.M.1384/1385.1	K -01-06111	28	0.58584
4078/1 519	4078/1 319	350	-1.07330	K -01-06111	K -01-07444	326	-0.30429
4078/1 319	4078/1 119	352	-1.06147	K -01-07444	K -01-07446	404	2.32751
4078/1 319	4078/1 119	352	-1.06183	K -01-07446	K -01-06802	427	1.08450
4078/1 319	4078/1 119	352	-1.06210	K -01-06802	K -01-07593	294	0.04873
4078/1 119	4078/919	353	0.50079	K -01-07593	K -01-07374	119	-0.44480
4078/1 119	4078/919	353	0.50024	K -01-07374	K -01-07592	329	0.18962
4078/1 119	4078/919	352	0.50042	K -01-07592	K -01-08007	339	-1.84751
4078/919	4078/719	351	1.04180	K -01-08007	K -01-08070	578	-3.48748
4078/919	4078/719	352	1.04195	K -01-08070	K -01-07536	232	3.40825
4078/919	4078/719	351	1.04157	K -01-07536	K -01-06024	198	5.73350
4078/719	4078/519	352	1.04383	K -01-06024	K -01-08191	362	1.84873
4078/719	4078/519	351	1.04365	K -01-08191	K -01-07535	340	0.70998
4078/719	4078/519	352	1.04360	K -01-07535	K -01-07103	590	3.28840
4078/519	4078/319	351	1.35766	K -01-07103	K -01-06876	278	-0.04820
4078/519	4078/319	351	1.35772	K -01-06876	K -01-07522	306	0.33439
4078/519	4078/319	352	1.35742	K -01-07522	K -01-07106	183	0.74556
4078/319	4078/119	352	4.71838	K -01-07106	K -01-06981	296	-0.06508
4078/319	4078/119	352	4.71759	K -01-06981	K -01-07594	315	-3.53902
4078/319	4078/119	352	4.71776	K -01-07594	1-13-09047	590	3.92598
4078/119	4078/100 502	290	6.17035	1-13-09047	1-13-09046	180	2.83377
4078/119	4078/100 502	288	6.16971	1-13-09046	1-13-09045	276	4.12991
4078/119	4078/100 502	289	6.17006	1-13-09045	1-13-09036	480	10.12574
4078/100 502	4078/100 501	107	2.80696	1-13-09036	1-13-09035	375	4.97433
4078/100 502	4078/100 501	106	2.80704	1-13-09035	1-13-09085	165	4.45871
4078/100 502	4078/100 501	107	2.80691	1-13-09085	1-13-09015	271	-0.22734
Copenhagen - Hamlets Grav				1-13-09015	G.I.1806	42	-3.35233
4078/100 501	92 164	807	5.43994	G.I.1806	G.I.1805	33	-0.69859
4078/100 501	92 164	811	5.43855	G.I.1806	G.I.1805	33	-0.69862
92 164	1-06-09129	608	1.25525	G.I.1805	1-02-09016	86	-0.52879
92 164	1-06-09129	607	1.25497	1-02-09016	1-02-09154	246	-3.34891
1-06-09129	1-06-09009	372	-2.84618	1-02-09154	1-02-09235	371	-4.69736
1-06-09129	1-06-09009	300	-2.84556	1-02-09235	1-02-09027	844	-4.96056
1-06-09129	1-06-09009	345	-2.84511	1-02-09027	1-02-09197	575	0.40066
1-06-09009	1-06-09130	322	0.29346	1-02-09197	1-02-09023	186	1.84284
1-06-09009	1-06-09130	329	0.29414	1-02-09023	1-02-09020	741	-3.76828
1-06-09009	1-06-09130	313	0.29329	1-02-09020	1-07-09059	364	0.80726
1-06-09130	1-06-09030	458	-0.42906	1-07-09059	1-07-09060	395	-1.35958
1-06-09130	1-06-09030	437	-0.42810	1-07-09060	1-07-09034	1271	-10.35894
1-06-09130	1-06-09030	410	-0.42850	1-07-09034	1-07-09005	278	1.13195
1-06-09030	1-06-09029	262	0.28163	1-07-09005	1-07-09004	956	13.25487
1-06-09030	1-06-09029	245	0.28100	1-07-09004	1-07-09061	199	-0.12945
1-06-09029	K -01-08160	939	-0.40423	1-07-09061	1-07-09062	423	2.01512
K -01-08160	K -01-07231	322	-0.82047	1-07-09062	1-07-09063	608	-0.32514
K -01-07231	K -01-07675	300	1.66577	1-07-09063	1-07-09066	205	-1.32782
K -01-07675	K -01-07736	431	-1.27456	1-07-09066	1-07-09064	219	-2.00371
K -01-07736	K -01-07800	342	1.21947	1-07-09064	1-07-09065	463	-4.81246
K -01-07800	K -01-07735	417	-0.45409	1-07-09065	1-14-09047	1122	21.27404
K -01-07735	K -01-07805	436	0.02328	1-14-09047	1-14-09048	640	-23.19757
K -01-07805	K -01-07833	373	-0.35008	1-14-09048	1-14-09003	668	15.81157
K -01-07833	K -01-07225	680	-0.86047	1-14-09003	G.I.2097	618	11.33078
K -01-07225	K -01-07733	262	1.89191	G.I.2097	1-14-09012	529	-0.63682
K -01-07733	K -01-07825	384	-0.60881	1-14-09012	1-14-09014	456	4.38710
K -01-07825	K -01-07559	507	0.17650	1-14-09014	13-01-09175	979	-4.89642
K -01-07559	K -01-08065	773	0.02748	13-01-09175	13-01-09176	869	-6.46304
K -01-08065	K -01-07769	612	0.12699	13-01-09176	13-01-09035	788	-0.49027
K -01-07769	K -01-08163	192	-0.07035	13-01-09035	13-01-09042	320	-3.92737
K -01-08163	K -01-06475	93	0.00546	13-01-09042	13-01-09055	403	0.40165
K -01-06475	K -01-07438	205	0.44600	13-01-09055	13-01-09155	1170	2.10255
K -01-07438	K -01-06817	612	0.78750	13-01-09155	13-01-09079	507	6.73383
K -01-06817	K -01-06429	193	-0.54556	13-01-09079	13-01-09157	764	-9.16765

13-01-09157	13-01-09178	722	9.13745
13-01-09178	13-01-09152	1055	-21.76584
13-01-09152	13-01-09068	895	7.78135
13-01-09068	13-03-09002	896	-4.22455
13-03-09002	13-03-09001	464	-2.71519
13-03-09001	13-05-09079	572	-3.37719
13-05-09079	13-05-09070	776	-1.59310
13-05-09070	13-05-00802	812	-0.27505
13-05-00802	13-05-09001	1226	-7.19419
13-05-09001	13-04-09010	1069	6.07794
13-04-09010	G.M.1330	483	0.18952
G.M.1330	13-04-09083	146	-0.64267
13-04-09083	13-04-09026	836	-2.65811
13-04-09026	13-04-09006	893	-16.91665
13-04-09006	13-04-09004	737	20.48722
13-04-09004	G.M.1332	938	-8.08294
G.M.1332	12-01-09154	1203	1.05930
12-01-09154	12-01-09155	1279	14.69375
12-01-09155	12-01-09156	919	11.36368
12-01-09156	G.M.1334	822	7.37570
G.M.1334	12-02-09010	722	1.49622
12-02-09010	12-02-09019	1139	-9.99446
12-02-09019	12-06-09058	641	-3.24903
12-06-09058	12-06-09113	116	-0.46502
12-06-09058	12-06-09113	99	-0.46534
12-06-09113	12-02-09016	602	-9.48240
12-02-09016	12-02-09008	873	12.68619
12-02-09008	12-02-09006	943	1.08087
12-02-09006	12-02-09033	822	3.40791
12-02-09033	12-02-09066	917	-4.44607
12-02-09066	K -06-09202	867	13.90090
K -06-09202	K -06-09166	1024	-20.86354
K -06-09166	K -06-09062	204	4.93669
K -06-09062	K -06-09138	983	-10.55822
K -06-09138	K -06-09064	442	-1.46610
K -06-09064	K -06-09072	733	2.22171
K -06-09072	G.I.1607	238	-5.87435
Kärnan: foundation wall - tower chamber			
032*1*3124	032*2*3109	40	6.98720

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